# Simple Methods for Approximating the Fractional Gradient Speeds of Photographic Materials*

C. N. Nelson and J. L. Simonds
*Research Laboratories, Eastman Kodak Company, Rochester 4, New York*
(Received August 12, 1955)

The American Standard method for determining the photographic speed of negative materials, and the corresponding British Standard, specify the 0.3 $\bar{G}$ fractional-gradient criterion for measuring speed sensitometrically. This criterion was adopted because it gives speed values which correlate more closely with speeds obtained by picture tests than do the speeds obtained by any other known sensitometric speed criterion. A frequent criticism, however, of the 0.3 $\bar{G}$ criterion is that it is more difficult to operate and more subject to random errors than simpler criteria, such as the one based on a density of 0.1 above fog. In the present paper, two methods of evaluating speed are described which are simple to operate and which give highly repeatable results that agree closely with accurately measured 0.3 $\bar{G}$ speeds and with speeds based on picture tests. The first method utilizes a mathematical representation of a portion of the sensitometric curve and converts inertia speed to an approximation of 0.3 $\bar{G}$ speed by means of a measurement of the length of the toe of the curve. The second method converts the speed based on a density of 0.1 above fog to an approximation of 0.3 $\bar{G}$ speed by means of the density resulting from an exposure equal to twenty times that required for the density of 0.1 above fog. Both methods are suitable for practical use, but the first method is of particular interest in the mathematical theory of the shape of the sensitometric curve and its relation to speed. The second method is especially suited to rapid, routine determination of speeds and is adaptable to automatic computation of speeds from density measurements without the necessity of drawing the sensitometric curve.

## INTRODUCTION

A FUNDAMENTAL definition of the speed of a negative photographic material for ordinary camera use is given in an American Standard,[1] a British Standard,[2] and various publications.[3–6] According to this definition, *speed* is inversely proportional to the minimum exposure of the negative material required to produce an excellent print of an average scene. Thus, the relative speeds of negative materials can be obtained by exposing the materials in a camera with progressively decreasing exposures, making the best possible print from each negative, and judging the prints to determine the least camera exposure that can be given without loss in print quality.

It is, of course, too time-consuming to use the print-judgment method except for basic studies. A sensitometric method is much more rapid and is, therefore, specified in the American[1] and British[2] standards for photographic speed determination. In this method, the 0.3 $\bar{G}$ fractional-gradient speed criterion is used. It was adopted because experimental studies[3–5] show that it gives speeds which correlate more closely with print-judgment speeds than do the speeds obtained by any other known sensitometric criterion. The inertia speed criterion was found to give poor correlation. Speeds measured at a density of 0.2 above fog were also unsatisfactory. Speeds measured at a density of 0.1 above fog were better than inertia or 0.2 density speeds, but did not correlate with the print-judgment speeds as closely as was desired for a primary standard method.

Figure 1 illustrates the method of applying the 0.3 $\bar{G}$ fractional-gradient criterion to the density-*versus*-log-exposure curve ($D$-log $E$) of the negative material. The speed value is equal to the reciprocal of the exposure, in meter-candle-seconds, at which the slope of the curve is three-tenths of the average gradient (called $\bar{G}$ or $\beta$), measured over a log exposure interval of 1.5 extending to the right of the speed point.

The good agreement between 0.3 $\bar{G}$ speeds and print-judgment speeds, and the lack of agreement found by the use of the fixed-density criterion, can be seen from the experimental results illustrated in Fig. 2. The six curves are density-*versus*-log-exposure curves for actual negative films. The three curves in the left-hand section of Fig. 2 represent films having differences in the length of the toe portion of the curve. The print-judgment

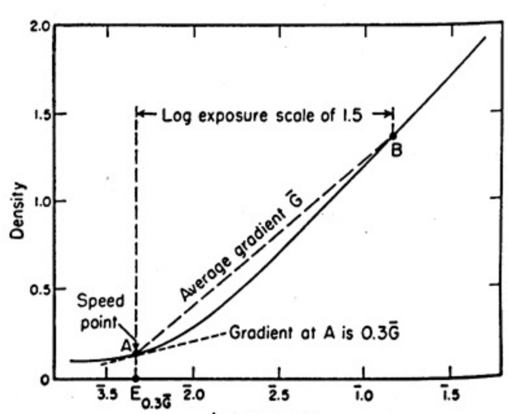


Fig. 1. Graphic method of determining fractional-gradient speed from the $D$-log $E$ curve.

* Communication No. 1741 from the Kodak Research Laboratories.

[1] American Standard Method for Determining Photographic Speed and Exposure Index Z38.2.1-1947, American Standards Association, 70 East 45th Street, New York 17, New York.

[2] British Standard Method of Determining Speed and Exposure Index of Photographic Negative Material, BS 1380.

[3] L. A. Jones, J. Franklin Inst. 227, 297, 497 (1939).

[4] L. A. Jones and C. N. Nelson, J. Opt. Soc. Am. 30, 93 (1940).

[5] L. A. Jones and C. N. Nelson, J. Phot. Soc. Am. 7, 10, 54 (1941).

[6] C. E. K. Mees, *The Theory of the Photographic Process* (The Macmillan Company, New York, 1942) first edition, p. 716; revised edition (1954), p. 877.

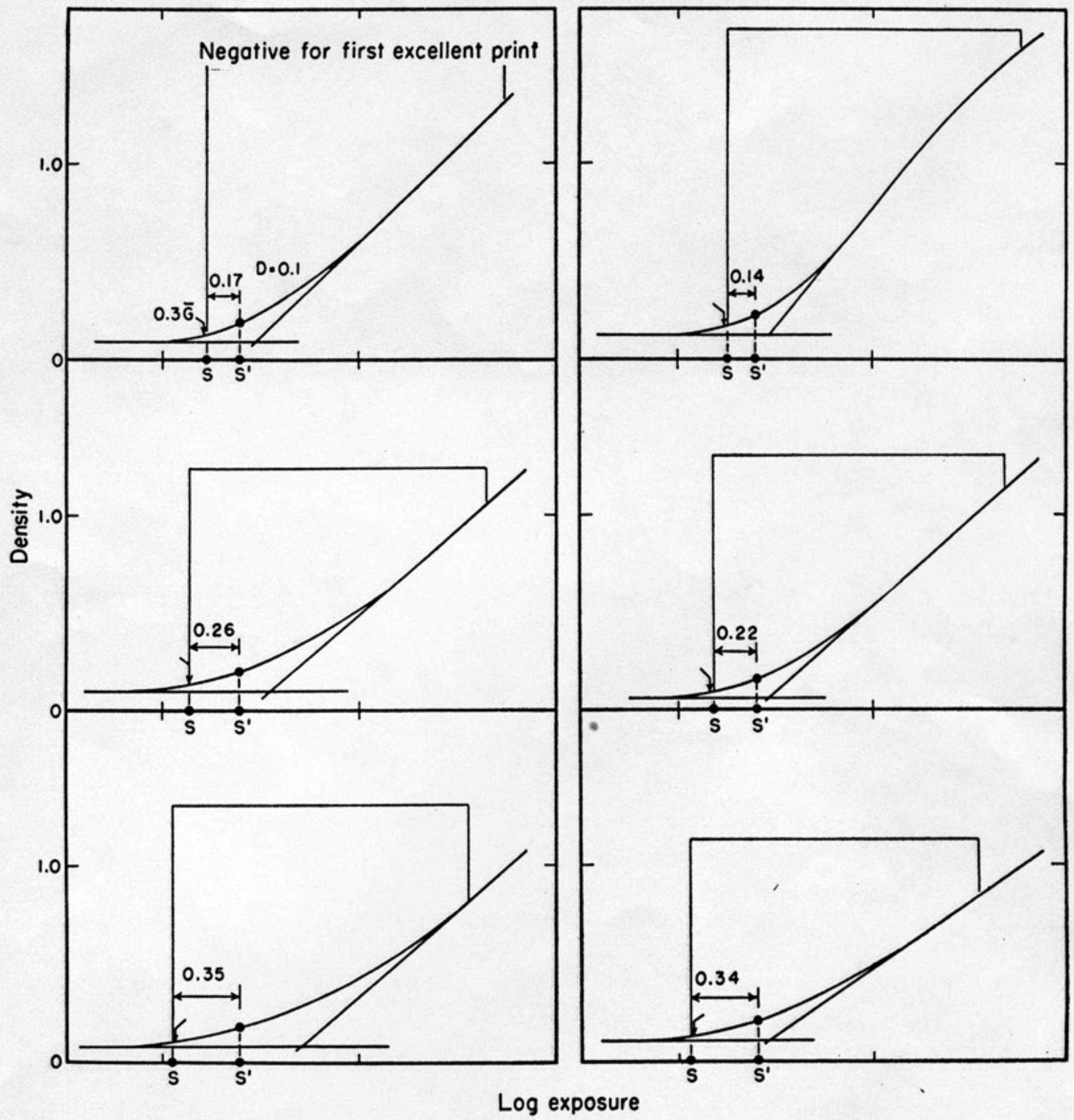


FIG. 2. Characteristic curves of actual negative films showing the inability of the 0.1 speed criterion to predict the effective speeds of films having different lengths of the toe portion of the curve and different slopes of the straight-line portion.

speed point, marked $S$ on the log $E$ axis for each curve, corresponds to the minimum exposure for the negative giving the "first-excellent" print, chosen by a number of observers. The 0.3 $\bar{G}$ fractional-gradient speed criterion gives the speed points indicated by the small arrows, which are seen to lie close to the print-judgment speed points, $S$. The 0.1 density criterion gives the speed points $S'$, which are seen to be displaced from the $S$ points by amounts which vary from 0.17 for a film having a short toe to 0.35 for a film having a long toe. Thus, the log $E$ error introduced by the 0.1 density criterion in comparing the speeds of the two films is 0.18, which is nearly two cube-root-of-two steps on the American Standard speed scale, or approximately two-thirds of a camera stop.

As shown in the right-hand section of Fig. 2, the 0.1 density criterion also gives an error as large as two cube-root-of-two steps (0.2 in log $E$) when negative materials are compared that differ in gamma. The error in the 0.3 $\bar{G}$ speeds is seen to be less than one cube-root-of-two step.

It is apparent also from Fig. 2 that the inertia point (the log $E$ at which the extension of the straight-line portion of the curve meets the line representing fog density) does not occur at a fixed interval from the print-judgment speed point. The size of the interval varies greatly for films differing in the length of the toe portion. Consequently, the inertia speeds are not satisfactory. From these and numerous other tests, it was concluded that the 0.3 $\bar{G}$ fractional-gradient criterion gives a more faithful indication of the effective speeds of the materials than is given by the other known criteria.

A frequent criticism of the 0.3 $\bar{G}$ criterion, however, is that it is more difficult to operate and more subject to random errors than simpler criteria, such as the one based on a density of 0.1 above fog. It is sometimes contended that the increase in ease and precision re-

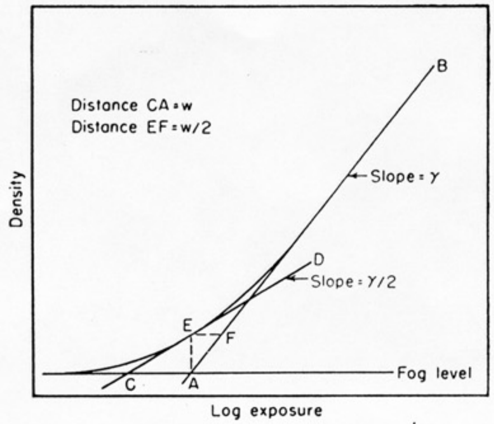


FIG. 3. Graphic method of determining the parameter, $w$, occurring in Luther's equation.

sulting from the use of the 0.1 density method outweighs the fact that such speeds do not correlate as well with print-judgment speeds.

A search has therefore been made for a criterion which would combine the ease and precision of the fixed-density speed measurement with the basic accuracy of the fractional-gradient speed criterion. An example of an attempt to obtain such a criterion was given in a recent paper by Frieser.[7] The paper offers an approximate means of transforming from a measurement of DIN[8] speed (0.1 density above fog) to fractional-gradient speed. The transformation proposed by Frieser is that, within certain limits on the length of the toe portion, the difference between the DIN speed point and the fractional-gradient speed point is a linear function of the slope of the straight-line portion of the characteristic curve. The theoretical basis of this transformation is that negative films possess characteristic curves which can be accurately approximated in the toe and straight-line portions by a mathematical equation given by Luther.[9]

In the present paper, two methods of speed evaluation are described which are simple to operate and which give highly repeatable results that agree closely with accurately measured 0.3 $\bar{G}$ speeds and with print-judgment speeds. The first method uses an extension of Luther's equation to predict the fractional-gradient speed point as a simple function of the inertia speed and one of the parameters, $w$, occurring in Luther's equation. The second method transforms the speed at a density of 0.1 above fog to an approximation of 0.3 $\bar{G}$ speed by means of the density resulting from an exposure equal to twenty times the exposure required for the density of 0.1 above fog. Both of the proposed methods provide speed values essentially identical with those which would be measured by the use of more difficult and less precise existing graphical metering procedures, without the limitations on length of toe imposed by the earlier approximation methods.

[7] H. Frieser, Phot. Korr. 90, 95 (1954).
[8] German Standard DIN 4512, adopted in 1934 and revised in 1954.
[9] R. Luther, Trans. Faraday Soc. 19, 340 (1923).

## USE OF LUTHER'S EQUATION TO PREDICT FRACTIONAL-GRADIENT SPEED

Several attempts have been made to approximate the shape of characteristic curves of photographic materials by mathematical functions. Luther's formula for the toe and straight-line portions is

$$D=\frac{\gamma w}{0.6}\log[10^{0.6(\xi/w)}+1], \qquad (1)$$

where $D$ is the density above fog, $\gamma$ is the slope of the straight-line portion, $w$ is the amount by which the log $E$ at the inertia exposure differs from the log $E$ at the intersection of the fog level and a straight line tangent to the curve at a point corresponding to the inertia exposure, and $\xi$ is equal to log exposure (corresponding to $D$) minus the logarithm of the inertia exposure.

Figure 3 illustrates these parameters. Line $AB$ is drawn tangent to the straight-line portion of the curve. The slope of $AB$ is equal to $\gamma$. The abscissa of the intersection of $AB$ with the line representing fog density is the inertia point. Line $CD$ is tangent to the curve at $E$, corresponding to the inertia exposure. It is a necessary consequence of Eq. (1) that the slope of $CD$ be equal to one-half the slope of $AB$. The parameter, $w$, is equal to the distance $CA$. It can also be shown that the distance $EF$ is equal to one-half of $w$.

TABLE I. Films and development conditions used in the present tests.

| Name of Kodak film used | Developer | Development time at 68 F (min) |
|---|---|---|
| Super-XX | DK-60a | 4 |
| Super-XX | DK-60a | 5 |
| Super-XX | DK-50 (1:1) | 3 |
| Super-XX | DK-50 (1:1) | 18 |
| Super-XX | DK-50 | 3 |
| Super-XX | DK-50 | 12 |
| Verichrome | ASA Dev. #1 | 3 |
| Verichrome | ASA Dev. #1 | 5 |
| Verichrome | ASA Dev. #1 | 8 |
| Royal Pan | DK-60a | 3 |
| Royal Pan | DK-60a | 4 |
| Royal Pan | DK-60a | 12 |
| Super Panchro-Press | DK-60a | 3 |
| Super Panchro-Press | DK-60a | 5 |
| Super Panchro-Press | DK-60a | 8 |
| Panatomic-X | D-76 | 5 |
| Panatomic-X | D-76 | 16 |
| Panatomic-X | DK-60a | 3 |
| Panatomic-X | DK-60a | 18 |
| Portrait Pan | ASA Dev. #1 | 5 |
| Super Panchro-Press | ASA Dev. #1 | 5 |
| Super-XX | ASA Dev. #1 | 5 |
| Ortho-X | ASA Dev. #1 | 5 |
| Panatomic-X | ASA Dev. #1 | 5 |
| Commercial | ASA Dev. #1 | 5 |
| Super Panchro-Press | DK-50 | 5 |
| Tri-X | DK-50 | 5 |
| Royal Pan | DK-50 | 5 |
| Super-XX Aero | D-19 | 5 |

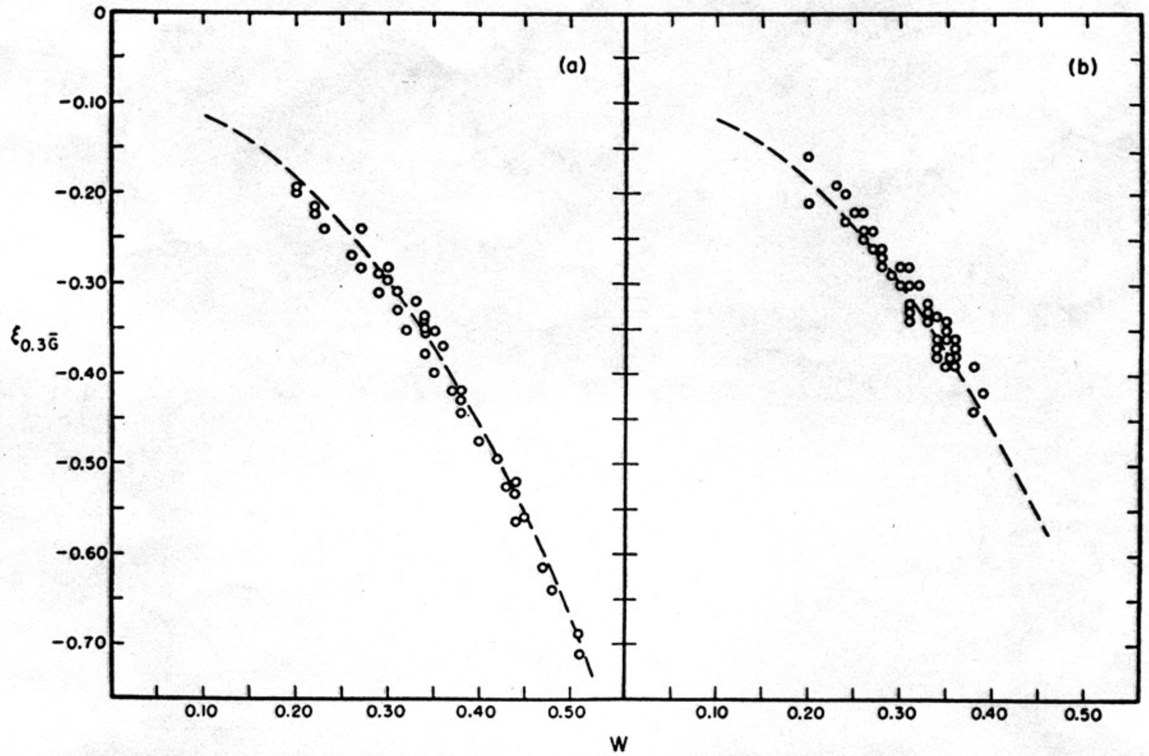


FIG. 4. (a) Plot of $w$ *versus* $\xi_{0.3G}$ for the forty negative films used in the original research on the 0.3 $\bar{G}$ criterion. (b) Plot of $w$ *versus* $\xi_{0.3G}$ for the film–developer–development-time combinations listed in Table I for current films. In both cases, the dotted curve is the least-squares parabolic fit to the combined data of both sets of films.

The slope of the characteristic curve at any point is given by

$$\frac{dD}{d\xi}=\gamma\frac{[10^{0.6(\xi/w)}]}{[10^{0.6(\xi/w)}+1]}. \tag{2}$$

At the inertia exposure, $\xi$ equals zero and Eq. (2) reduces to $\gamma/2$.

Equations (1) and (2) can be used to determine the relation among the fractional-speed point, $w$, and gamma. This relation is

$$\tfrac{1}{2}w[10^{0.6(\xi_{0.3G}/w)}+1]\log\frac{[10^{0.6(\xi_{0.3G}+1.5)/w}+1]}{[10^{0.6(\xi_{0.3G}/w)}+1]}-10^{0.6(\xi_{0.3G}/w)}=0, \tag{3}$$

where $\xi_{0.3G}$† is the log exposure difference between the 0.3 $\bar{G}$ speed point and the inertia point. It can be seen from Eq. (3) that $\xi_{0.3G}$ is independent of gamma and can be computed from a simple measurement of $w$.

A solution of Eq. (3) for $\xi_{0.3G}$ as a function of $w$ does not result in a simple functional relation between the two parameters. The following procedure was used to determine a simple means of transforming from $w$ to $\xi_{0.3G}$.

† The symbol 0.3 $\bar{G}$ is, for typographical reasons, replaced by 0.3 $G$ when it is used as a subscript in this text. Since 0.3 $\bar{G}$ is often used in photographic literature, it is preferred whenever it can be used conveniently.

Sensitometric curves were obtained for each of the forty films used in Jones and Nelson's original research on the fractional-gradient speed criterion. On each curve, accurate measurements of the fractional-gradient speed point and the inertia exposure were made. The following two measures of $w$ were also made: (a) Referring to Fig. 3, $w$ is measured as twice the distance $EF$, or; (b) a straight line of slope $\gamma/2$ is moved parallel to the log $E$ axis until it is tangent to the characteristic curve. The difference between the log $E$ at the inertia point and the log $E$ of the intersection of the straight line of slope $\gamma/2$ with fog is a measure of $w$.

If Luther's equation is a perfect representation of the characteristic curves, the two measures of $w$ will be identical. The two values of $w$ for each of the forty films were not significantly different. Method (a) is the simpler of the two and therefore is to be preferred.

Figure 4(a) is a plot of $\xi_{0.3G}$ *versus* $w$ for each of the forty films. Figure 4(b) is a similar plot of $\xi_{0.3G}$ *versus* $w$ for each of the film–developer–development-time combinations listed in Table I. These films are representative of current negative films. Examination of both sets of data indicates that the relation between $\xi_{0.3G}$ and $w$ can be approximated by an equation of the form

$$\xi_{0.3G}=a+bw+cw^2. \tag{4}$$

The constants $a$, $b$, and $c$ were obtained by a least-squares computation, based on the data presented in

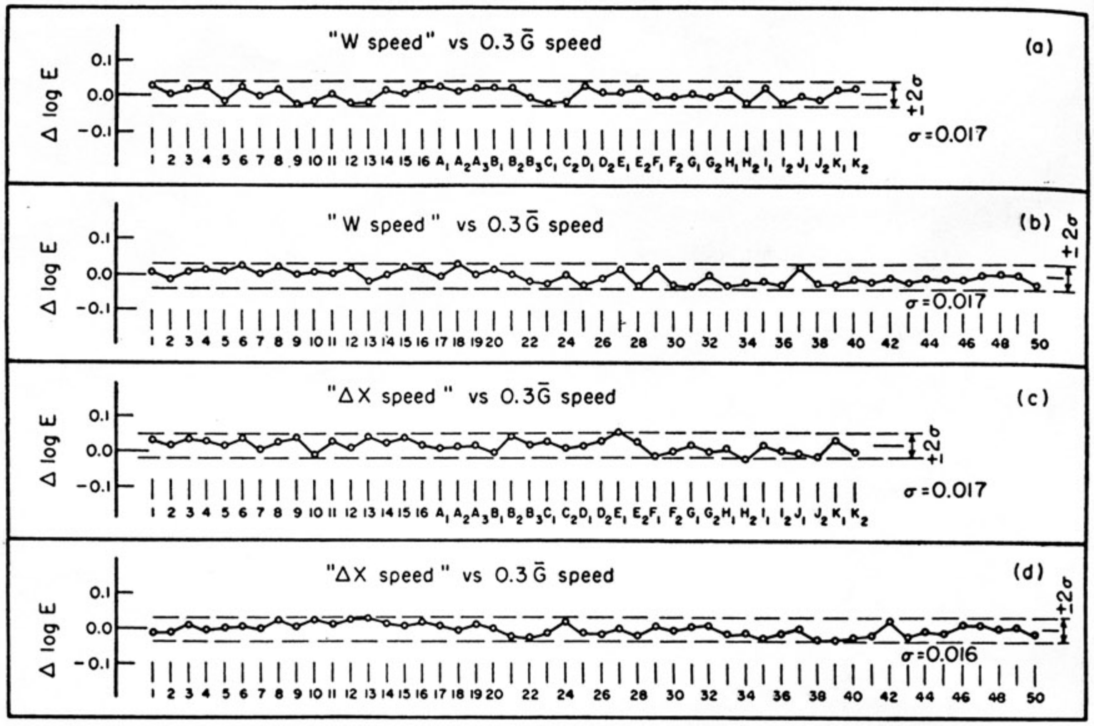


FIG. 5. Logarithmic differences between the speeds determined by various methods for (a) and (c), the films used in the original research on the 0.3 $\bar{G}$ criterion, and for (b) and (d), the current films listed in Table I.

both Fig. 4(a) and Fig. 4(b). The resulting least-squares relation is

$$\xi_{0.3G} = -0.0948 + 0.0122w - 2.2945w^2. \quad (5)$$

The dotted curve in Figs. 4(a) and 4(b) is the least-squares parabola of Eq. (5).

For each of the films tested, a calculation was made of the difference between the speed points determined by a measurement of $w$ and subsequent application of Eq. (5) and the accurately measured fractional-gradient speed points. Figure 5(a) is a plot of this difference for each of the forty films of Jones and Nelson. Figure 5(b) is a plot of this difference for the films and development combinations listed in Table I. In no case does the difference exceed $\pm 0.03$ in log exposure. The standard deviations of the differences are indicated on both charts.

In Fig. 6, comparisons are made of the various measures of speed and the print-judgment speeds determined by Jones and Nelson on their original forty films. Figure 6(a) is a plot of the log exposure resulting in a density of 0.1 above fog minus the log exposure of the negative giving the first-excellent print. Figure 6(b) is a similar comparison made of fractional-gradient speed points and the print-judgment speed points. Figure 6(c) is a plot of the differences of the speed points calculated by the "$w$ method" and the print-judgment speed points. The speeds calculated by the "$w$ method" correlate as well with print-judgment speeds as do the speeds determined by the fractional-gradient criterion.

The ability to predict $\xi_{0.3G}$ from a measurement of $w$ makes possible a meter designed for rapid measurement of $w$ and the fractional-gradient speed point. Figure 7 is a diagram of the two sections of the proposed meter to be printed onto a transparent base. The two parts are to be riveted together at the points marked "pivot" so that one may rotate with respect to the other about the pivot point.

Figure 8 shows the meter being applied to a sensitometric curve. The vertical line of Fig. 7(a) is to be placed parallel to the density axis at the log exposure of the inertia point. The horizontal line of 7(a) is then placed at a density equal to the density of the characteristic curve about the inertia point. Section 7(b) is rotated until line $AB$ passes through the intersection of the horizontal line of 7(a) and the extension of the straight-line portion of the characteristic curve. Line $CD$ on section 7(b) has been so drawn that, when the above settings have been made, the horizontal line of section 7(a) will intersect with line $CD$ at the log exposure of the fractional-gradient speed point. The form of line $CD$ depends on the placement of the pivot point. Given a specific location of the pivot point, line $CD$ can be determined empirically.

This meter offers a simple and rapid means of speed-point specification based on the relation between $w$ and $\xi_{0.3G}$. An additional feature of this meter is that

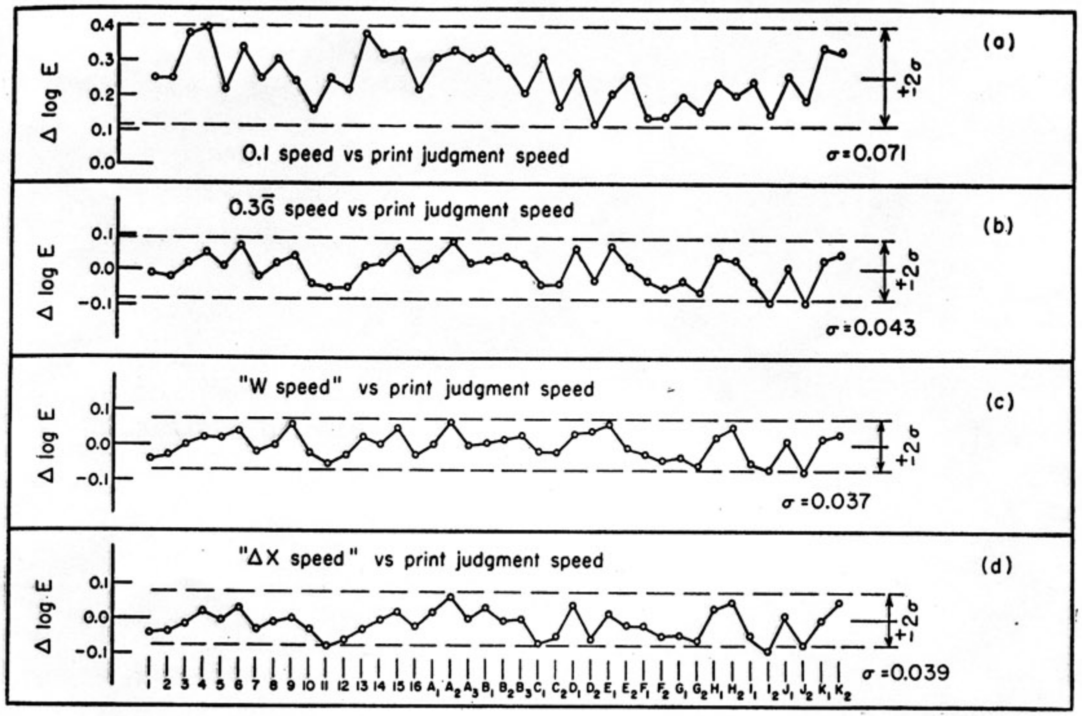


FIG. 6. Logarithmic differences between the speeds determined by various methods and the speeds determined by print-judgment techniques for the films used in the original research on the 0.3 $\bar{G}$ criterion.

the value of $w/2$ can be read directly from the scale provided on section 7(a).

### DELTA *X* SPEED CRITERION

Another speed criterion, which is especially advantageous from the standpoint of ease of operation, is based on the finding that the DIN speed criterion ($D$=0.1 above fog) can be used as a starting point to obtain a close approximation of the 0.3 $\bar{G}$ fractional-gradient speed and the speed based on the least exposure of the negative material which will result in a print of excellent quality. The speed obtained by the proposed approximation method is called "delta $X$ speed." The transformation from the speed at a density of 0.1 above fog to delta $X$ speed can be accomplished by simple graphic or computational methods involving only two points on the $D$-log $E$ curve: (1) the exposure, $E$, required for a density of 0.1 above fog, and (2) the density obtained from an exposure which is twenty times greater than $E$.

The derivation of the delta $X$ speed criterion is illustrated in Figs. 2, 9, 10, and 11. From Fig. 2 and from similar data for other films, it can be seen that the log exposure difference ($\Delta X$) between the 0.3 $\bar{G}$ speed point and the density of 0.1 above fog is small when the average gradient or "$\beta$ value" of the first-excellent negative is large, and is large when the average gradient is small. Thus, there seems to be a reciprocal or inverse relation between the log exposure difference and the average gradient. A similar relation is seen to exist between $\Delta D$ and $\Delta X$, as illustrated in Fig. 9. The density, $D_1$, equal to 0.1 above fog is indicated by a black dot on the $D$-log $E$ curve. At a log $E$ value lying 1.3 to the right of this point, a density, $D_2$, is found on the curve. An interval of 1.3 was chosen in order that $D_2$ would be approximately equal to the highlight density in the first-excellent negative of an average scene. The difference, $D_2-D_1$, is called $\Delta D$. This density difference, $\Delta D$, is an approximate measure of

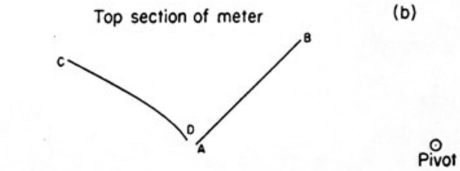


FIG. 7. Components of a meter designed for rapid determination of "$w$ speed" from characteristic curves. Both sections are to be printed on a transparent base and riveted together at the pivot point.

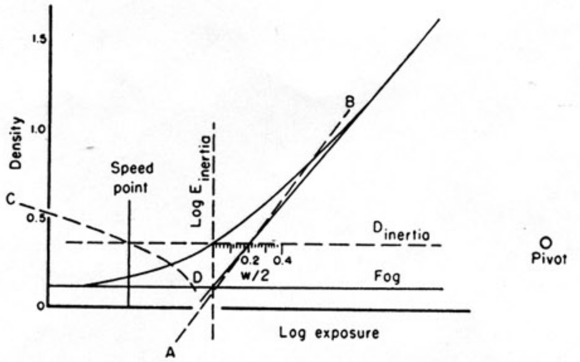


FIG. 8. The "*w* speed" meter applied to a sensitometric characteristic curve. The dotted lines indicate the markings on the meter sections.

the average gradient of the portion of the curve used to obtain a first-excellent negative. Consequently, it was recognized that $\Delta D$ might be a reliable means of predicting the logarithmic difference, $\Delta X$, between the speed at a density of 0.1 above fog and the fractional-gradient speed. As shown in Fig. 9, when $\Delta D$ is large, $\Delta X$ is small; when $\Delta D$ is small, $\Delta X$ is large.

The relation between $\Delta D$ and $\Delta X$ was, therefore, measured for the forty $D$-log $E$ curves of the negative materials used in the original derivation of the 0.3 $\bar{G}$ fractional-gradient speed criterion.[2] The results are shown in Fig. 10(a). Similar measurements were carried out for a number of current films in several different developers. The values are given in Fig. 10(b). It is

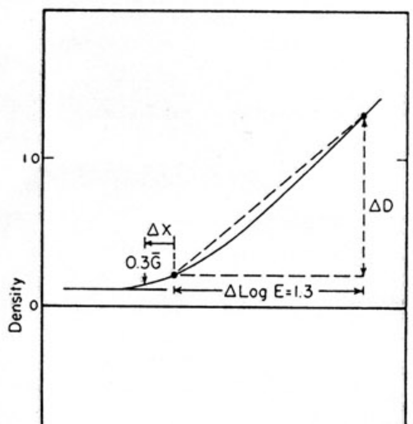


FIG. 9. Demonstration of the inverse relationship which exists between $\Delta D$ and $\Delta X$. The density equal to 0.1 above fog is indicated by the black dot on the characteristic curve. $\Delta D$ is measured at a log exposure equal to the log exposure of the 0.1 point plus a $\Delta$ log $E$ of 1.3.

seen that a definite relation between $\Delta D$ and $\Delta X$ does exist. The curved line drawn through the plotted points is the same for both sets of data. This line is a parabola, fitted to the data by the method of least squares. It represents the data points very well and it can, therefore, be used as a means of approximating fractional-gradient speed from 0.1 density speed. Because of the potential usefulness of this average relationship between $\Delta X$ and $\Delta D$, a table of values of $\Delta X$ and $\Delta D$ was prepared, based on the curve of Fig. 10. These values are given in Table II. This table can be used in conjunction with the $D$-log $E$ curve of any negative material as a means of converting 0.1 density speed to delta $X$ speed. Also, this table can be used in constructing a meter, such as that shown in Fig. 11, which can be applied to

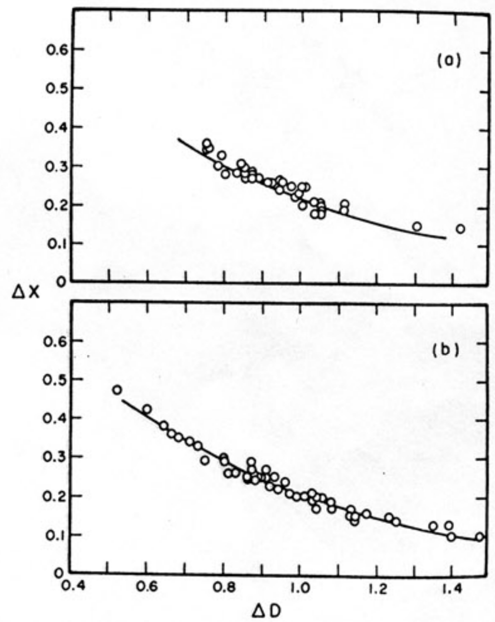


FIG. 10. (a) $\Delta X$ *versus* $\Delta D$ for the films used in the original research on the 0.3 $\bar{G}$ criterion. (b) $\Delta X$ *versus* $\Delta D$ for the current films listed in Table I. In both graphs, the curve shown is the least-squares parabolic fit to the data for the current films.

the $D$-log $E$ curve to perform very quickly the transformation of 0.1 density speed to delta $X$ speed.

The equation for the curve in Fig. 10 is as follows:

$$\Delta X = 0.83 - 0.86\Delta D + 0.24\Delta D^2. \quad (6)$$

This equation represents a parabola and is of interest in the problem of calculating speed values by means of an automatic computing device which can be designed to use the density readings obtained directly from an automatic densitometer.

By means of a meter of the type shown in Fig. 11, delta $X$ speeds were determined for the forty $D$-log $E$ curves used in the original derivation of the 0.3 $\bar{G}$ fractional-gradient speed criterion. The logarithmic differences between the delta $X$ speeds and the 0.3 $\bar{G}$

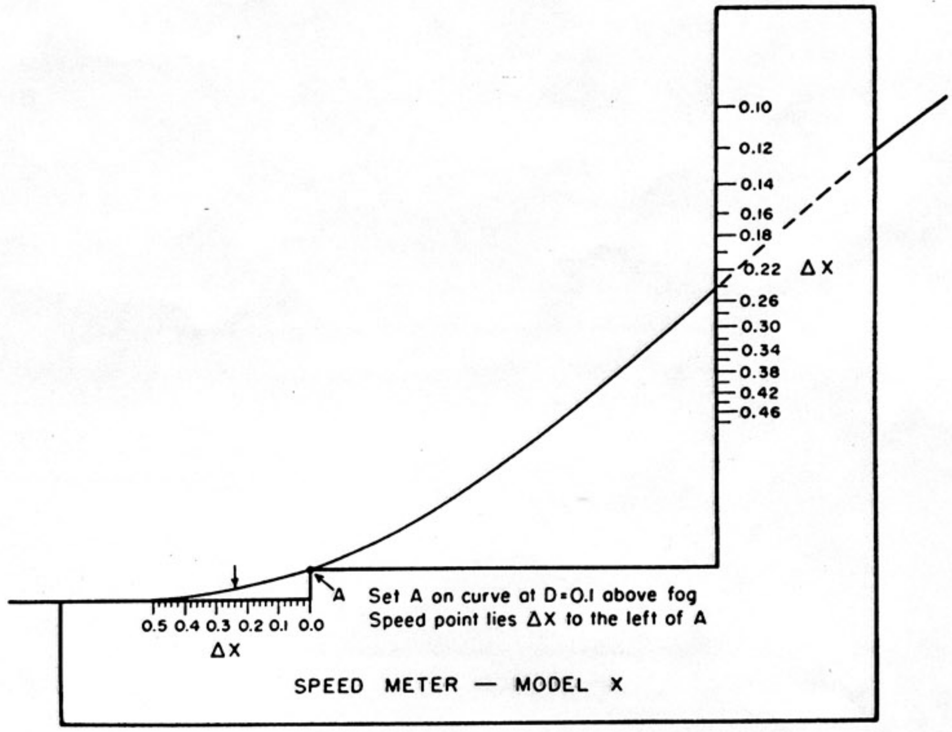


FIG. 11. The meter designed for rapid determination of "$\Delta X$ speed" from characteristic curves. Point $A$ is set on the curve at $D=0.1$ above fog. The $\Delta X$ value is read from the vertical scale. The speed point lies $\Delta X$ units of log $E$ to the left of point $A$.

speeds are shown in Fig. 5(c). The differences are found to be very small. The standard deviation is 0.017. The forty curves used in this test represented a number of films of several manufacturers, the films being processed in several different developers. A further comparison of delta $X$ and 0.3 $\bar{G}$ speeds was made using films of current production, also with several different developers. The results are given in Fig. 5(d). The differences are again very small, the standard deviation being 0.016. The conclusion is that the delta $X$ speeds are a very satisfactory approximation of the 0.3 $\bar{G}$ speeds.

A comparison was also made of delta $X$ speeds and the print-judgment speeds reported by Jones and Nelson.[3] The log differences are shown in Fig. 6(d). The standard deviation is 0.039. Thus, the delta $X$ speeds correlate as well with the print-judgment speeds as do the speeds determined by the 0.3 $\bar{G}$ fractional-gradient criterion, the standard deviation for the latter being 0.043.

## PROCEDURES FOR DETERMINING DELTA $X$ SPEEDS

The delta $X$ speed, $S_x$, is defined by the expression,

$$S_x = 1/E_x, \tag{7}$$

where the value of $E_x$ is obtained from the relation,

$$\log E_x = \log E_{D=0.1} - \Delta X. \tag{8}$$

$E_{D=0.1}$ is the exposure in meter-candle-seconds required to obtain a density of 0.1 above fog, and $\Delta X$ is the $\Delta \log E$ "correction increment." As mentioned in connection with the derivation of the delta $X$ speed criterion, $\Delta X$ depends on $\Delta D$, in accordance with Table II.

The delta $X$ speed can, if necessary, be determined from the $D$-log $E$ curve without the aid of any special meter or computing device. The procedure is as follows: A density, $D_1$, equal to 0.1 above fog, is located on the

TABLE II. The average relation between $\Delta D$ and $\Delta X$, based on the data of Fig. 10.

| $\Delta D$ | $\Delta X$ | $\Delta D$ | $\Delta X$ |
|---|---|---|---|
| 0.50 | 0.46 | 1.00 | 0.21 |
| 0.52 | 0.45 | 1.02 | 0.20 |
| 0.54 | 0.44 | 1.04 | 0.20 |
| 0.56 | 0.43 | 1.06 | 0.19 |
| 0.58 | 0.41 | 1.08 | 0.18 |
| 0.60 | 0.40 | 1.10 | 0.18 |
| 0.62 | 0.39 | 1.12 | 0.17 |
| 0.64 | 0.38 | 1.14 | 0.17 |
| 0.66 | 0.37 | 1.16 | 0.16 |
| 0.68 | 0.36 | 1.18 | 0.15 |
| 0.70 | 0.35 | 1.20 | 0.15 |
| 0.72 | 0.33 | 1.22 | 0.14 |
| 0.74 | 0.32 | 1.24 | 0.13 |
| 0.76 | 0.31 | 1.26 | 0.13 |
| 0.78 | 0.30 | 1.28 | 0.12 |
| 0.80 | 0.29 | 1.30 | 0.12 |
| 0.82 | 0.28 | 1.32 | 0.11 |
| 0.84 | 0.27 | 1.34 | 0.11 |
| 0.86 | 0.26 | 1.36 | 0.11 |
| 0.88 | 0.25 | 1.38 | 0.10 |
| 0.90 | 0.25 | 1.40 | 0.10 |
| 0.92 | 0.24 | 1.42 | 0.10 |
| 0.94 | 0.23 | 1.44 | 0.09 |
| 0.96 | 0.22 | 1.46 | 0.09 |
| 0.98 | 0.22 | 1.48 | 0.09 |

curve and the log $E$ at this point is recorded. At a log $E$ position lying 1.3 to the right of this point, a second density value, $D_2$, is obtained from the curve. Subtracting $D_1$ from $D_2$ gives $\Delta D$, which is converted to a $\Delta X$ value by means of Table II. Subtracting $\Delta X$ from log $E_{D=0.1}$ gives log $E_x$, the antilogarithm of which is then used in the relation, $S_x=1/E_x$, to calculate the delta $X$ speed.

A more convenient and rapid procedure is to use a meter of the type shown in Fig. 11. The point, $A$, on the meter is placed on the $D$-log $E$ curve at a density of 0.1 above fog. The horizontal edges of the meter are kept parallel to the horizontal lines of the graph paper. A value of $\Delta X$ is read from the upper (vertical) scale on the meter at the point where the $D$-log $E$ curve intersects this scale. The delta $X$ speed point, log $E_x$, is then marked on the graph paper at the log $E$ position lying $\Delta X$ to the left of $A$, the lower (horizontal) scale of $\Delta X$ being used to locate the log $E_x$ point. From the log $E_x$ value, the speed, $S_x$, is calculated from the relation, $S_x=1/E_x$.

A meter of this type can be made on photographic film, sheet plastic, sheet metal, or even cardboard. A trial model of the meter was made photographically as a transparency positive. An accurate ink drawing of the meter was first prepared which was one and one-half times larger than the final meter. The drawing was photographed on a negative material suitable for line reproduction, using the proper reduction in size so that the meter would fit the graph paper on which the sensitometric curves are ordinarily plotted. From the negative, a number of the meters were made on film by contact-printing. Practical tests show that this meter operates satisfactorily. Meters of different sizes are required for graph papers of different scale sizes.

The delta $X$ method is of particular interest in the problem of evaluating photographic speed rapidly and precisely by means of an automatic computing device attached to an automatic densitometer. The delta $X$ criterion is much more adaptable to this type of operation than the 0.3 $\bar{G}$ criterion. From the density readings made on the exposed and processed sensitometric strip of the negative material and from the exposure calibration of the sensitometer, a suitably designed computer can determine the log exposure required for a density of 0.1 above fog, obtain $\Delta D$ from the density readings, solve Eq. (6) for $\Delta X$, and compute the final speed value.

### PRECISION OF THE PROPOSED METHODS OF SPEED DETERMINATIONS

For ten negative films, digital density and log $E$ values (for 0.15 log exposure intervals) were furnished to eighteen operators. These operators were experienced in drawing accurate sensitometric curves from digital data. Each operator was asked to plot the data and draw each of the ten sensitometric curves. On each

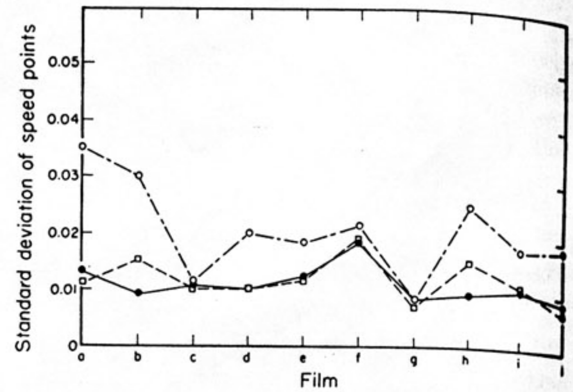


FIG. 12. Data on the repeatability of speed measurements by three methods: —·—0.3$\bar{G}$, – – – $\Delta X$, and ——— $w$. The standard deviations of speed (in terms of log $E$) are plotted for eighteen operators and ten films.

of these 180 curves, the following measures of speed were made: (a) fractional-gradient speed by the use of the Model C Speed Meter,[10] (b) speed determined by a measurement of $w$ and inertia speed and conversion to $\xi_{0.3G}$, (c) speed determined by the delta $X$ method.

For each film, the standard deviations of the speed points by the three methods were determined. Figure 12 is a plot of the standard deviations (in log $E$ units) of the speed points. The small circles show the standard deviations obtained with the fractional-gradient method. The small squares apply to the delta $X$ method. The black dots apply to the $w$ method. In each case, the standard deviations of the speeds obtained by the two approximation methods are equal to, or less than, the standard deviations of the fractional-gradient speeds measured by the Model C gradient meter. Thus, both approximation methods yield more repeatable results than the direct method of measuring 0.3 $\bar{G}$ speeds by means of the fractional-gradient speed meter.

The fractional-gradient meter is, of course, still regarded as a basic and useful device for obtaining reference values of 0.3 $\bar{G}$ speed. Such speeds can be determined precisely by averaging the values obtained with several samples of each negative material. Care must be taken to determine the $D$-log $E$ curves accurately for each sample. The American Standard specifies that twelve samples of each negative material be used to obtain the speed value. When this is done, the precision of the final result is, of course, very satisfactory. The merit of the proposed approximation method lies mainly in the ease and rapidity with which a precise result can be obtained. This advantage is particularly important in testing large numbers of different films for which it is sometimes impractical to use more than a few samples.

A useful feature of the proposed methods is that they avoid the necessity of drawing accurately the low-

[10] L. D. Clark, PSA Journal (Phot. Science and Tech.) 17B, 87 (1951).

gradient portion of the toe of the $D$-log $E$ curve. Experience shows that the 0.3 $\bar{G}$ speed is particularly sensitive to errors in drawing the lower part of the toe.

## DEVELOPMENT CONDITIONS

In the experimental work reported [3–5] in this country on the 0.3 $\bar{G}$ fractional-gradient speed criterion, the development conditions used for the picture negatives were always the same as those used for the sensitometric samples. In extensive tests made in Europe and reported by Eggert[11] and Hiltpold,[12] 0.3 $\bar{G}$ speeds and 0.1 density speeds obtained with a standard developer and standard processing conditions were compared with print-judgment speeds obtained with different developers and different processing conditions in various laboratories. The differences between the print-judgment speeds for the various processing conditions were large compared with the differences between the 0.3 $\bar{G}$ and 0.1 density speeds. It was concluded that the 0.3 $\bar{G}$ and 0.1 density criteria were not significantly different in their capacity to predict the print-judgment speeds for development conditions other than those used in evaluating the sensitometric speeds. This conclusion is sometimes interpreted too broadly as an indication that there is no significant difference between the two criteria.

It is important to note that the experimental results[11,12] obtained in Europe do not contradict the experimental results[3–5] obtained in this country and summarized in Fig. 2 of the present paper. It is well known that the speed of a film will, in general, change if a different developer or different degree of development is used. The change is sometimes large and sometimes small, and may be either negative or positive, but it can be measured by applying the 0.3 $\bar{G}$ criterion to each of the $D$-log $E$ curves for the different processing conditions. The several values of speed thus obtained for any given film may differ considerably from each other; and the single value obtained with the standard processing conditions will not necessarily equal the average of the different values obtained with the other processing conditions.

In the tests reported by Eggert[10] and Hiltpold,[11] the 0.3 $\bar{G}$ criterion and the 0.1 density criterion were applied only to the curves for the standard processing conditions. It is to be expected that neither of the two criteria can give an accurate indication of the speeds for the other processing conditions. As a consequence, the distinction between the 0.3 $\bar{G}$ and 0.1 density criteria is obscured in such a test. From one point of view, it can be argued that an accurate speed criterion is unnecessary, since no criterion can take into account the variations in processing that may occur in practical pictorial photography. On this basis, the simple and convenient fixed-density criterion is sometimes considered to be adequate.

The point of view in the standardization work in this country has been, however, that it is often desirable to be able to predict the effective speed of a film for specified processing conditions. For this purpose, the 0.3 $\bar{G}$ criterion is the most accurate, especially if the 0.3 $\bar{G}$ speed values for several samples of each film are averaged to minimize the random errors. Moreover, this criterion is no worse than other criteria with respect to its inability to predict, without direct tests, the speeds for other processing conditions. The best that can be done toward the solution of the problem of the effect of development differences is to specify, for use in the standard method of evaluating speed, the processing conditions which most closely match the average conditions found in practice.

The disadvantage of following this viewpoint has been that the speed criterion required is more complex and therefore more difficult to apply. The simplified criteria proposed in the present paper overcome this disadvantage. They are accurate indicators of print-judgment speeds when applied to tests using development conditions the same as those used in deriving the sensitometric curves, and are also as accurate as any other known speeds in the problem of estimating, from a sensitometric test, the average print-judgment speeds for development conditions other than those used for the test. It is hoped that these new criteria will provide a basis for more complete agreement on the standardization of methods of determining photographic speed.

## STANDARDIZATION

The proposed methods of speed measurement may be of interest to committees concerned with preparing or revising standards for determination of photographic speed. The 0.3 $\bar{G}$ fractional-gradient criterion probably should be retained as the primary or fundamental means of speed determination, since it is based on tone-reproduction principles which make it especially suitable for evaluating any new type of film that might be designed having an unusual $D$-log $E$ curve shape. The proposed simplified methods appear to be satisfactory, however, for films normally encountered in practice. Consequently, it seems appropriate to recommend their adoption in speed standards. The standards could, for example, specify that one or both of the simplified methods be permitted as an alternative to the primary method.

## ACKNOWLEDGMENT

The writers are indebted to J. L. Tupper for initiating this project and contributing a number of helpful suggestions and also to F. C. Williams for valuable counsel.

[11] J. Eggert, Schweiz. Photo-Rundschau 193 (1948).
[12] R. Hiltpold, Z. wiss. Phot. 47, 189 (1952).