The Incident Light Method of Exposure Determination

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ABSTRACT. The performance of the flat plate or cosine type incident light receptor is compared with that of the hemispherical or cardioid type, and values are suggested for the photographic constants when these two types of receptor are used. After an introductory description of the techniques of exposure determination the difference in performance of the two types of incident light receptor are deduced by consideration of their angular response characteristics in relation to particular lighting conditions. Experimental evidence is given in support of the theoretical considerations.

1. METHODS OF EXPOSURE DETERMINATION

1.1. The Requirement

PHOTOGRAPHIC processes may be considered to be of two kinds. First there is that by which the exposed film is converted by chemical means into a negative and a separate printing process is then used to obtain the final photograph. This is the negative/positive process which may result in monochrome or colour pictures. Secondly, there is the process by means of which the exposed film is converted into the final photograph, a transparency, without the intermediary of a separate negative. This is the reversal process and likewise may result in monochrome or colour pictures. Because the former technique achieves its objective by two separate processes compensation can be made for incorrect exposure of a negative, to some extent, by variation of the printing conditions. For reversal work no such adjustment is possible and in consequence the exposure required for a given subject must be made with greater accuracy when the reversal process is used, than when the negative/positive process is used.

Initially, for monochrome negative/positive work, the exposure required for satisfactory photography of a given subject was determined by assessment of the light reflected from the darkest subject shadows which it was required to reproduce. Provided that the range of brightness of the scene is equal to or less than the possible range of density of the negative or print, this method gives satisfactory results. If the range of brightness of the scene is greater than that which can be reproduced, this method would result in over-exposure of the highlights of the scene. The next development was, therefore, to determine the exposure by measurement of the brightest part of the scene and to allow under-exposure in the shadows. This is generally more acceptable than over-exposure of the highlights because the more interesting part of a scene is usually that which is the better illuminated, and because in motion picture work the eye has not the time to investigate the shadows, so that loss of detail in the darkest parts is not usually a serious disadvantage. Further, in colour work the photographic requirement is that the brighter parts shall be adequately reproduced, and these are, therefore, of relatively greater significance in exposure determination.

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In practice the techniques mentioned above would appear to require the direct measurement of the light from either the darkest shadow or the brightest surface. This could be effected by the use of a meter with a very small acceptance angle which would be directed towards and receive light from only the shadow, or only the highlight. The light readings thus obtained would, by means of a calculator mechanism, be translated into a setting of camera aperture and shutter speed suitable to place the darkest shadow at the lowest usable part of the exposure/density curve of the film used, or the highlight at the highest part. The S.E.I. photometer[1] is an instrument of this kind.

An exposure meter with a small acceptance angle necessarily requires a photo-sensitive element of high sensitivity because of the small amount of light energy received in this small angle. This, and its consequential effect on the construction of the photometer, militates against the widespread use of this type of meter.

1.2. The Reflected Light Method

Exposure meters most commonly encountered have a much larger acceptance angle than that of the S.E.I. photometer; it is of the same order of magnitude as that of a camera. These meters receive light from the whole of a scene; it is not limited to either the darkest or the brightest parts. Their indication is, therefore, a measure of the total light received from the scene within its acceptance angle and this indication is related to the indication which would be obtained from the shadows, in the same way that the average brightness of the scene is related to that of its darkest significant shadow; significant, that is, because the detail in this shadow is required to be reproduced in the photograph.

This technique will be seen to relate the required exposure, not to the lowest permissible part of the exposure/density curve, but to a point above the lowest point, chosen by photographic tests to prove satisfactory for the majority of scenes photographed[2]. It relies on a measure of integrated scene brightness and because this is measured by light reflected from the scene, the method is called the reflected light method of exposure determination. Despite the fact that the relationship between the integrated brightness and the brightness of the darkest significant shadow is actually far from being a constant quantity, experience and practice over many years have shown that the reflected light method, when properly applied, can give satisfactory photographic results.

It is to be noted that the calculator mechanism providing information about relative aperture and shutter speed for a meter with a very small acceptance angle is not the same as it must be for a meter with an acceptance angle approximately equal to that of the camera, for the former must relate to either the darkest shadow or the brightest highlight (or both) whilst the latter is related to a middle tone.

1.3. The Incident Light Method

The exposure meter used for the reflected light method of exposure determination could also be used for the second technique mentioned previously, that is, the measurement of the highlight of the scene. In this case the brightest part of the scene would be approached closely so that the part included within the acceptance angle of the meter would be only the brightest part; alternatively a white card could be used in a similar manner, to simulate it. In this case the calculator mechanism would relate the meter indication to the highest part of the exposure/density curve.

It is from this second technique of measurement that the incident light method was evolved.

This is based on the fact that the brightness of a particular part of a scene bears a specific relationship to the light incident upon it, so that the incident light can be taken as a measure of the reflected light provided a suitable factor is introduced to take account of the reflectivity of the subject. The brightness of the brightest part of a scene cannot exceed that which would be obtained with the given incident intensity normal to a surface having 100 per cent reflectivity. If then a measurement is made of the maximum incident light, by directing the meter incident light receptor from the subject position towards the source of maximum illumination, and the calculator mechanism is arranged to relate this illumination to the highest part of the exposure/density curve, then, because the reflectivities normally encountered are less than 100 per cent. all scenes having this incident light will, in theory be reproduced satisfactorily on the sensitive film, working from the brightest to the darkest with which the contrast range of the film can deal[3]. However, bearing in mind the condition that the required camera exposure is dependent upon the image illumination within the camera, and a measure of this quantity is, therefore, necessary, it will be evident that the brightness which it is required to measure is that apparent from the camera position.

The measurement of the illumination incident upon the receptor must, therefore, take account of the fact that the illumination on a surface normal to the camera subject axis is less than the maximum possible, if the source-subject axis is at an angle (other than zero) to the camera-subject axis. To allow for this the incident light receptor should apparently be directed from the subject position towards the camera.

It may happen that the two incident light measurements so made are the same; this will occur when the maximum source is behind the camera, but if, as is usual, they are not, a conflict of requirements arises.

Should the incident light receptor be directed from subject to source or subject to camera? Either method gives incorrect results when the source-subject axis is at a considerable angle to the camera-subject axis.

It is to be noted that the meter indication need not be a measure of illumination expressed in any ordinarily accepted units. It is merely a means to provide adjustment of the calculator mechanism; thus the calibration level of the meter is immaterial provided that of the calculator is adjusted to be in agreement with that of the meter. This adjustment can only be effected by determining by photographic tests, the conditions necessary to give satisfactory photographs.

Thus if the incident light receptor absorbs a certain percentage of the incident light it can give satisfactory exposure indication provided that the calculator is adjusted to put this percentage of the light at a correspondingly lower point on the exposure/density curve, leaving the maximum brightness at the highest part. It would appear obvious so to adjust the absorption of the incident light by its receptor that the required adjustment to the calculator mechanism would result in the same mechanism being equally applicable to either the reflected or incident light method of exposure determination. This process is not quite the same as occurs with the reflected light method which, as has been stated, relates the integrated brightness to a middle or average tone. In this case the indication remains related always to the incident light and is independent of the subject except in so far as the subject itself modifies the incident light. The calibration level may, for convenience but not by necessity, be related to that for the reflected light method of measurement.

To revert to the question of the method of measurement. It has been found by photographic tests that the logarithmic average of the meter readings obtained by the two methods, source to subject and camera to subject provides more satisfactory exposure information than either taken separately.

Finally, it is to be noted again that the validity of any method of exposure determination must be proved acceptable or otherwise by photographic tests.

1.4. The Photographic Equations

For the reflected light method the conditions for correct photographic exposure* have been condensed into an equation, the photographic equation[4,5] which is given in all exposure meter Standards in one form or another and can be expressed as

$$K=\frac{LSt}{A^2} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

where K is a constant determined by photographic tests.

L is the integrated brightness of the scene measured over the acceptance angle of the exposure meter.

S is the film speed.

t is the exposure time (shutter speed).

and A is the relative aperture (*f*/number).

The existence of this photographic equation for the reflected light method of exposure determination has stimulated the formulation of a similar equation for the incident light method.

This can be given as:—

$$C=\frac{ESt}{A^2} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

where C is a different constant determined by photographic test.

E is the incident illumination

and the other symbols have the same significance as previously.

It will be noticed that in equation (1) the significance of the symbol L is clearly and precisely defined, but that in equation (2), E is not precisely defined, for no indication is there given of the direction from which the incident light emanates, nor the relationship between this direction and the influence of the light on the meter. This present lack constitutes a deficiency in the specification of the incident light method.

In the course of development of the method two forms of light receptor have been suggested and used; the first consisted of a flat plate of translucent material and the second of a hemisphere of translucent material[6]. These have different response characteristics.

It is the main purpose of this paper to discuss the spatial pick-up characteristics of these two types of receptor, the value of the constant C of equation (2) applicable to each, and to suggest that the hemispherical receptor has more suitable characteristics than the flat plate receptor for exposure determination by the incident light method.

2. TYPES OF LIGHT RECEPTOR FOR INCIDENT LIGHT MEASUREMENT

In order to gain some precision in the following discussion, the characteristics of each of the two types of incident light receptors will be expressed in the mathematical terms to which their optical performance most nearly approximates, and the practical conditions of use will be approached by consideration first of the extreme conditions which can be postulated, and secondly by modification of these conditions in such a way that they not only become closer to the

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*The definition of correct exposure in this context is intended to be that given in Jones & Condit's writings on photographic exposure:—"The correct camera exposure is, therefore, the *least* which will yield a negative from which an excellent print can be made".

practical conditions but also lend themselves to mathematical treatment.

2.1. The Flat Plate Receptor

If the illumination on a flat surface produced by light falling normal to the surface is E then the illumination produced by the same source when the surface is inclined at angle α to the direction of the light is E cos α. The response of a meter fitted with a flat plate receptor can then be represented by

$$R_b = k_b \, E \cos \alpha \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

where the factor k_b takes into account the relationship between the illumination on the receptor and the reading of the meter. R_b is thus the arithmetic value of the meter reading.

Hence the flat plate receptor may be described as a cosine type receptor, and this term will now be used in this discussion. It is to be clearly understood that in practice the flat plate receptor supplied with an exposure meter seldom has a pick-up characteristic which can be represented perfectly by the above equation. Specular reflection from its polished surface and the shading effects of the meter case both cause departure of performance from the mathematical model. This, however, is not necessarily of such significance that the use of equation (3) becomes invalid.

2.2 The Hemispherical Receptor

The characteristics of this type of receptor have been discussed elsewhere in considerable detail[7] and it has been shown that it can be represented by an equation of the type

$$R_a = \tfrac{1}{2} k_a \, E(1 + \cos \alpha) \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

in which the symbols have a similar significance to those given in equation (3). This is the equation of a cardioid, and the hemispherical type receptor will now be described as of the cardioid type.

The use of the cardioid type receptor is based on the fact that the normal subject for photography is three dimensional, and therefore, can reflect towards the camera light coming from any direction in space, except that immediately behind it. The simplest three dimensional shape which can be considered to represent the three dimensional subject is the hemisphere, for it has facets oriented in all directions in space from which light can be reflected towards the camera.

3. METHOD OF USE OF THE COSINE AND CARDIOID TYPE RECEPTORS

3.1. The Cosine Type Receptor

The cosine type receptor was intended originally to be directed from the subject position towards the source of maximum illumination, on the basis that the highlight of the scene is determined by the maximum value of the incident light[3]. Later considerations suggest that it should be directed from the subject to the camera, and it has already been explained that in either case the meter indication is not satisfactory under all conditions of illumination, for unless the lighting condition and method of use are the same as those for which the meter was calibrated a significant error occurs in exposure determination.

To overcome this defect the cosine type receptor in use is directed both to the camera and to the maximum source from the subject position and an average of the two readings so obtained is taken. This is known as the Duplex method of exposure determination[3]. Because an exposure meter is calibrated in terms of stops and these represent a logarithmic progression, the average mentioned above is the logarithmic average of the two readings taken. Thus if R_{bs} and R_{bc} are the source direction and camera direction readings respectively, the reading used to determine exposure is

$$R_b = \sqrt{R_{bs} \times R_{bc}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

it being remembered that the values of R_b are the arithmetic values of meter response. In practice the meter is calibrated in divisions corresponding to the logarithm of R so that the logarithmic average is simply half way on the meter scale between the two readings obtained. This is a very simple figure to find from the meter scale.

When using a cosine type receptor, the duplex method represents so considerable an advance in exposure determination, over either the source direction or the camera direction methods that it will henceforth be assumed that it is the only method to be considered further.

3.2. The Cardioid Type Receptor

The cardioid type receptor is developed on the basis that it represents in its simplest form a three-dimensional subject having facets oriented in any direction in space from which light can be reflected towards the camera. Because of this it is intended to be placed in the subject position and directed towards the camera. It is, of course, permissible to place it in some other position where the incident light is the same and direct it in a line parallel with that between subject and camera, so that the same reading is obtained. This is merely a matter of convenience for the subject position may be inaccessible to the photographer.

It is contended that the use of a cardioid type receptor is superior to that of a cosine type receptor because (i) it gives a better representation than the cosine type of the photographic conditions encountered when the subject is three-dimensional, (ii) it permits determination of exposure by a single reading of the meter, whereas the use of the cosine type receptor requires two readings and the determination of an average.

It should be clearly understood here that there is not intended to be any implication that the cardioid type receptor is "right" whilst the cosine type receptor is "wrong". Either can be and has been used for exposure determination by experienced photographers with success. The contention merely is that of the two the cardioid type is the better choice for the reasons given.

4. ESTIMATION OF THE PERFORMANCE OF INCIDENT LIGHT RECEPTORS

The derivations of performance obtained in the paper are based on mathematical models intended to simulate practical conditions. This is an idealized concept in which the conditions of the model are not actually identical with the conditions encountered in practice and this fact must be borne in mind when comparing the theoretical and practical conditions. Nonetheless it will be observed that the calculations made from the theoretical postulates have been expressed to at least three significant figures. This has been done because the model is expressible in precise terms and no advantage accrues by adopting approximations too drastic or too early in the argument, which might introduce irregularities in calculated values.

It will subsequently be seen that wide differences between theoretical and practical values are not considered to vitiate the validity of the arguments.

It may be presumed that if the incident light measurements are valid means for exposure determination then the same reading must be obtained under any given set of lighting conditions when using either the cosine type or the cardioid type receptor. The comparison between the two types, therefore, resolves itself into finding the optical conditions of the receptors for the meter to give the same readings.

The optical constants k_a and k_b can be related to the photographic constant C for each of the two types of receptor.

The meter reading is a function of the product of a constant k and the incident light, and can be represented by

$$R = k\,f(E)$$

where f (E) results from the duplex or single process of measurement. For a given light condition, and distinguishing between the two types of receptor by suffices a and b, we have

$$f(E_a) = \frac{R_a}{k_a} \text{ for the cardioid receptor}$$

$$\text{and } f(E_b) = \frac{R_b}{k_b} \text{ for the cosine receptor.}$$

Also the photographic equation is

$$C_a = \frac{f(E_a)\,St}{A^2} \text{ for the cardioid receptor}$$

$$\text{and } C_b = \frac{f(E_b)\,St}{A^2} \text{ for the cosine receptor}$$

$$\text{whence } \frac{C_a}{C_b} = \frac{f(E_a)}{f(E_b)} = \frac{R_a}{k_a}\,\frac{k_b}{R_b}$$

and when as is required the two readings are equal for the same incident illumination

$$R_a = R_b \text{ and } \frac{C_a}{C_b} = \frac{k_b}{k_a} \qquad \ldots \quad \ldots \quad \ldots \quad (6)$$

4.1. Calculation Under Extreme Lighting Conditions

There are two extremes of lighting conditions possible; these are:—

(i) that there is a point source, and no other source of illumination at all.

(ii) that there is a completely uniform source of brightness.

Neither of these conditions is met in normal photographic practice, but it is of interest to note that both can be obtained fairly easily in the laboratory, and the first is actually that which is used in the calibration of an exposure meter. This condition can be obtained on the photometric bench in which all light except that from a standard lamp is effectively screened by baffles from the exposure meter. The second condition is obtained in an integrating sphere.

Normal lighting conditions are a mixture of the two extremes. One or more point sources usually exist and there is more or less uniform lighting from the surroundings.

When there is a single point source the cosine type receptor will give a reading $R_{bs} = k_b E$ when directed to the source, a reading $R_{bc} = k_b E \cos\alpha$ when directed to the camera. The reading to be used in the determination of exposure is then

$$R_{b1} = \sqrt{R_{bs} \times R_{bc}} = k_b E \sqrt{\cos\alpha}$$

The cardioid receptor will give a reading

$$R_{a1} = \tfrac{1}{2} k_a E(1 + \cos\alpha)$$

and for these two readings to be the same

$$R_{a1} = R_{b1}$$

$$\text{therefore } k_b E\sqrt{\cos\alpha} = \tfrac{1}{2} k_a E(1+\cos\alpha) \qquad \ldots \quad (7)$$

There is obviously no single value of k_a and k_b which meets this requirement, the ratio k_a/k_b is a function of $\cos\alpha$, further if α is equal to or greater than 90° the cosine type receptor gives zero response whilst the cardioid receptor still gives a reading. This factor points out the limitation of the cosine type receptor, for it is quite evidently possible to

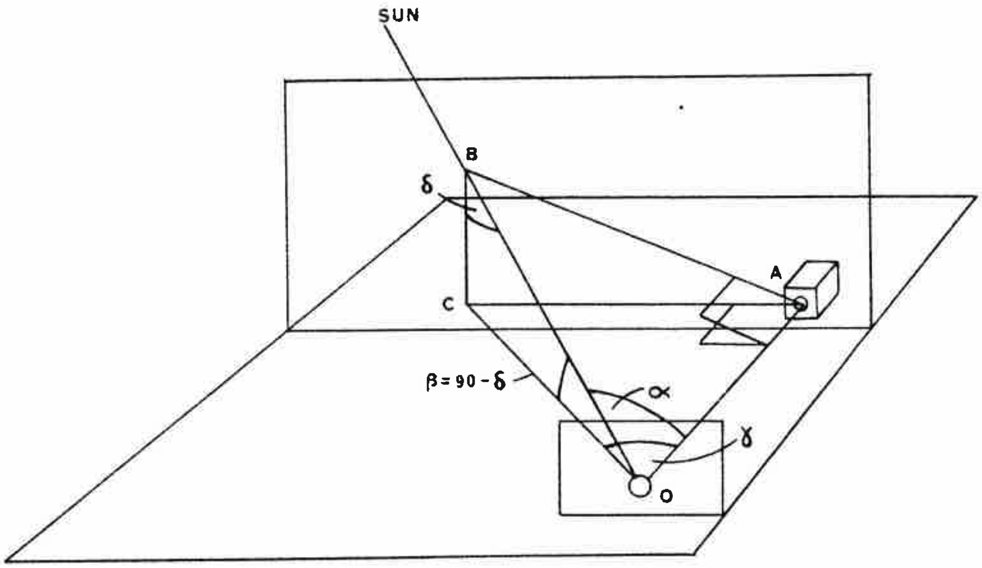

Fig. 1. Sun in a clear sky.

take a photograph when the light is incident at angles greater than 90° and up to, but not including 180°, and the cosine type receptor gives no information about this condition.

If the simple condition is taken of the light behind the camera and falling fully on the subject, then $\alpha = 0$ and equation (7) then shows that $k_a = k_b$. From equation (6) $C_a = C_b$.

A complication is introduced into this argument by the fact that the meter is calibrated logarithmically not arithmetically and therefore, has no true zero on its light scale. Nonetheless the pointer giving light indication must take up a position of rest somewhere over the scale when no light falls on the meter, and this appears to be a zero. Thus the meter can apparently give a logarithmic average reading even when one of its readings should theoretically be zero as, for instance, when α exceeds 90°.

The conditions which arise when the receptor is placed in a completely uniform source of brightness L, are investigated in detail in Appendices A and B where it is shown that the response of a cosine type receptor to the uniform brightness obtained in an integrating sphere is

$$R_{b2} = \pi\ k_b\ L$$

whilst that of a cardioid type receptor is

$$R_{a2} = 2\ \pi\ k_a\ L$$

If these two readings are to be equal

$$k_b = 2k_b \text{ and } \frac{C_b}{C_a} = \frac{k_a}{k_b} = \tfrac{1}{2}$$

that is, the light transmission of the cardioid receptor needs to be one-half of that of the cosine receptor.

The two extremes of illumination conditions

$$\frac{C_b}{C_a} = 1 \text{ and } \frac{C_b}{C_a} = \tfrac{1}{2}$$

are thus seen to make different demands on the optical properties of the receptors and to provide

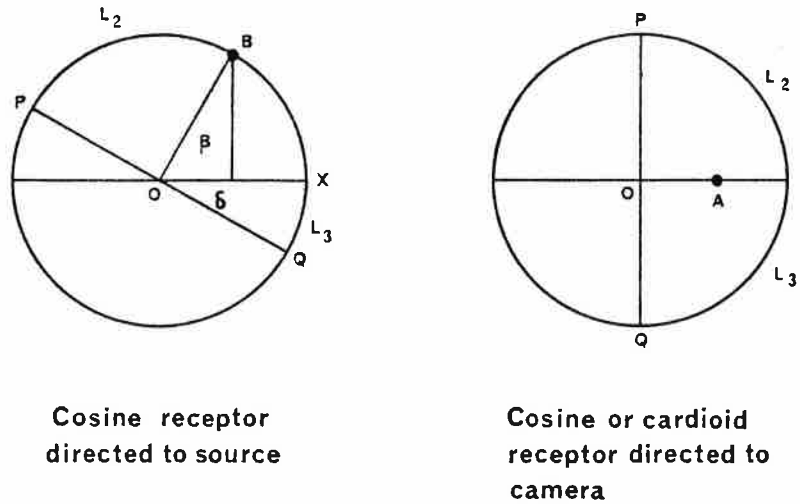

Fig. 2. Method of use of the cosine receptor.

satisfactory performance in ordinary use some compromise is necessary. This compromise is approached in the next section by consideration of the conditions which could occur in normal photography.

4.2. Estimation under Natural Lighting Conditions

The diversity of light and colour proportions and distribution which have to be dealt with in photography is so great that a precise mathematical treatment of the incident light method of exposure determination which will take all cases into account is obviously impossible. It is, however, possible by using, in the right proportions, the extreme conditions already investigated to build up a reasonable estimate of some of the conditions which can occur naturally.

For instance the sun may be represented by a single point source of light at a certain elevation and angle relative to the subject/camera axis and a clear sky may be represented by a hemisphere of uniform brightness. The earth may be considered as an infinite plane of uniform brightness different from that of the sky. Further, an infinite plane can be represented by a hemisphere. Each of these conditions can be represented by mathematical expressions dependent upon the observed brightness of sun and sky, and average reflectance of the earth.

Values can be assigned to these quantities and thereby the response of an exposure meter can be estimated for conditions which represent only small modifications of the normal conditions of photography. This has been done in detail in Appendices C and D, but will be summarized here.

4.2.1. *Sun in a clear blue sky*

The conditions to be considered are described in the terms shown in Fig. 1. The reference plane containing camera A and subject O is assumed to be horizontal. The illumination incident upon the subject is a maximum in the direction BO from the point source at B. The relative positions of source, subject and camera are defined by the angles α β δ and γ, where α is the angle of incidence of the light on the subject relative to the subject/camera direction, β is the sun elevation, δ the angle such that $\beta+\delta=90°$ and γ the angle between the projection CO of the line BO on the horizontal plane and the line OA. γ is hereafter called the "sun angle".

A cosine type receptor is pointed from the subject position first towards the maximum source, i.e., along the line OB, and secondly towards the camera, i.e., along the line OA. Refer to Fig. 2.

A cardioid type receptor is pointed only along the line OA.

A cosine type receptor directed to the maximum source will have light incident upon it from:—

(i) the point source (the sun) at B along the receptor axis.

(ii) the blue sky represented by the upper hemisphere through B of brightness L_2 between the planes POQ and OX.

(iii) the earth represented by the lower hemisphere between the planes OX and OQ, of brightness L_3.

It is shown in Appendix D that the meter reading would then be:—

$$R_{bs}=(11\cdot 95+1\cdot 425 \cos \delta-0\cdot 897 \cos^2 \delta)\ k_b$$

A cosine type receptor directed from the subject to the camera will have light incident upon it from:—

(i) the point source at an elevation $\beta=90-\delta$.

(ii) the blue sky represented by one half of the upper hemisphere of brightness L_2.

(iii) the earth represented by one half of the lower hemisphere.

The meter reading under this condition (see Appendix D) would then be:—

$$R_{bc}=0\cdot 729+11\cdot 219 \sin \delta \cos \gamma+0\cdot 897 \cos \delta)\ k_b$$

The value used for exposure determination

$$R_b=\sqrt{R_{bs}\times R_{bc}}$$

Representative values of δ and γ can be chosen and the response R_b of the cosine type receptor can be calculated. This has been done and the figures are shown in Table I.

Table I

Figures proportional to meter readings using the cosine type receptor in conditions of sun and clear blue sky

Sun angle	0	30	60	90	120	150	180
Sun elevation	Figures proportional to meter reading.						
$\beta=90-\delta=90$	4·50	4·50	4·50	4·50			
60	9·43	8·92	7·34	4·34			
45	10·78	10·14	8·14	4·11	Same as for 90°		
30	11·67	10·95	8·70	3·91			
0	11·95	11·17	8·77	2·95			

A cardioid type receptor directed from the subject to the camera is shown in Appendix D (taking into account the assumptions made in Appendix C) to give correspondingly a reading

$$R_a=(7\cdot 07+1\cdot 79 \cos \delta+5\cdot 61 \sin \delta \cos \gamma)\ k_a$$

Table II shows values of R_a for representative values of δ and γ.

Table II

Figures proportional to meter readings using the cardioid type receptor in conditions of sun and clear blue sky

Sun angle	0	30	60	90	120	150	180
Sun elevation	Figures proportional to meter reading.						
$\beta=90-\delta=90$	8·86	8·86	8·86	8·86	8·86	8·86	8·86
60	11·43	11·05	10·02	8·62	7·22	6·19	5·82
45	12·30	11·77	10·32	8·34	6·35	4·90	4·37
30	12·82	12·17	10·39	7·96	5·54	3·76	3·11
0	12·68	11·92	9·87	7·07	4·26	2·21	1·46

4.2.2. *Completely overcast sky*

In this condition there is no point source of illumination, but the brightness of the sky can be considerably higher than that of the clear blue sky. The reflexion from the earth is assumed to be the same percentage of the incident illumination as it was previously. The condition is investigated in Appendix D where it is shown that the meter readings are:—

$R_b=0\cdot 957\ k_b\ q$ for a cosine type receptor.

$R_a=1\cdot 458\ k_a\ q$ for a cardioid type receptor.

the factor q, common to both measurements takes into account the greater brightness of cloud relative to that of the blue sky. Since our objective is to determine the relationship between the constants k_a and k_b and thence C_a and C_b, the numerical value of q is fortunately not required to be known.

5. COMPARISON OF THE TWO RECEPTORS UNDER NATURAL CONDITIONS OF ILLUMINATION

It has already been pointed out that under extreme conditions of illumination the ratio $\frac{C_b}{C_a}$ may require to be either unity or one half and that some compromise is necessary. The natural conditions for which meter readings have been estimated approach the extremes of those likely to be encountered in normal photography, and for these the ratio $\frac{C_a}{C_b}=\frac{k_b}{k_a}$ can be found by comparison of the figures in Section 4.

5.1. Sun in a Clear Blue Sky

This comparison is made in Table III derived from Tables I and II, in which is shown the ratio k_a/k_b for each of the chosen values of sun elevation and sun angle on the assumption that k_a and k_b have been so chosen that the meter reading will be the same for a given light condition with both of the receptors.

This is obtained for the first figure in the tables by equating $4{\cdot}50\ k_b$ to $8{\cdot}86\ k_a$, whence $\frac{k_a}{k_b}=\frac{4{\cdot}50}{8{\cdot}86}=\frac{C_b}{C_a}$ and implies that the cosine type receptor must have a greater light transmission, a lower constant C_b than that, C_a, for the cardioid type receptor.

Table III

Comparison of the constants $\frac{C_b}{C_a}$ of cosine and cardioid type receptors

Sun angle	0	30	60	90	120	150	180
Sun elevation	Ratio $\frac{k_a}{k_b}=\frac{C_b}{C_a}$						
$\beta=90$	·51	·51	·51	·51			
60	·83	·81	·73	·50	Not valid		
45	·88	·86	·79	·49			
30	·91	·9	·84	·49			
0	·94	·94	·89	·42			

It is difficult to choose an acceptable value of the ratio C_b/C_a from this tabulation because of the wide variation it shows. It is, therefore, decided that an average of all values shown is a fair compromise for receptors which may be used under any possible lighting condition. On this basis the ratio $\frac{C_b}{C_a}=0{\cdot}71$.

5.2. Completely Overcast Sky

There is for this condition one figure only for each of the receptors, and the ratio is then:—

$$\frac{k_a}{k_b}=\frac{\cdot 96}{1\cdot 46}=0\cdot 66$$

It will be seen that this does not differ greatly from the figure found for the previous conditions, and as a first estimate it can be expected that a ratio $\frac{C_b}{C_a}=0{\cdot}7$ will be satisfactory for most photographic work.

This estimate must, of course, be verified by photographic tests, and is discussed further in a later section.

5.3. General Conditions

It is evident that the usual conditions met with in photography will be somewhere between the two dealt with in the previous two sections. The sun is not often seen in a completely blue sky without haze or some cloud, nor does the presence of an overcast sky mean that the sky and earth are of uniform brightness. On the other hand the departure from these conditions is not particularly serious in most cases, except where the reflectance of the earth is very high, such as occurs when snow has fallen, or the scene is of a beach of sand or shells. In this case the conditions approach the uniformly bright sphere, not the hemisphere. These conditions are special, and for such, special precautions must be taken in translating the light measurement into terms of the required exposure.

5.4. Experimental Evidence

5.4.1. *Method of Test*

The performance of the receptors can be checked relatively to each other without taking photographs, by observing the meter reading obtained with each type of receptor for the same scene and also by measuring for each, on a photometric bench, its constant C and pick-up characteristic. An average difference obtained from a number of observations taken in differing circumstances can be found and used to apply a correction to the constant of either one of the receptors in order to bring its average performance into line with that of the other. It will then be found that the constants C_a and C_b are not equal and their ratio can be compared with the estimated value already given.

A test of this nature has been carried out and to ensure that the conditions of test approximated as nearly as possible to the conditions assumed in the estimate made previously, observations were taken in the middle of a large recreation ground where the ground was virtually the "infinite" plane assumed, and days with clear skies and bright sun were chosen as far as possible. It was not possible to obtain readings with a completely clear sky, nor was it possible to obtain a sun elevation greater than that permitted by the latitude of the observer's location and the time of year.

Readings were taken using the cosine type receptor, first with its axis pointed directly to the sun and then horizontally for sun angles at 30° intervals round the circle. Readings were also taken using a cardioid receptor for the same horizontal orientations of its axis.

In addition to the above, a set of readings was obtained using the cosine receptor and the cardioid receptor in conditions of nearly uniform illumination, obtained in the same location as above but on a day which was heavily and uniformly overcast, and when the meter indication was very little, if at all affected

Table IV
Comparison of cardioid and cosine type receptors (sun in a blue sky)

1	2	3	4	5	6	7	8	9	10	11	12
Type of receptor Constant C			*Cardioid* 40	*Cosine* 51						*Ratio* C_b/C_a *estimated for* $\delta=54°$	*Col.* (10) *-Col.* (11)
δ	γ		*Reading*	*Reading*	*Average*	*Col.* (4) *-Col.* (6)	*Factor*	C_1	*Col.* (9) / 40		
		$a=0$	—	13/1½							
54	0		13/1½	12/2	12/4¾	2¾	1·37	37·2	·930	·85	1·09
	30		13/1	12/1	12/4¼	2¾	1·37	37·2	·930	·83	1·12
	60		13/0½	11/4	12/2¾	3¾	1·54	33·1	·827	·75	1·10
	90		13/0	10/4	11/5¾	6¼	2·06	24·8	·620	·50	1·24
	270		13/0	10/3	11/5¼	6¾	2·18	23·4	·585	·50	1·17
	300		13/1	11/1½	12/1½	5½	1·88	27·1	·677	·75	·90
	330		13/1½	11/5½	12/3½	4	1·58	32·3	·807	·83	·97
	360		13/2	12/1½	12/4½	3½	1·50	34·0	·850	·85	1·00
							Average		·778	·732	1·064

by the horizontal direction in which it was directed, i.e., it was independent of γ.

5.4.2. *Results of Tests*

Several observations of the kind described above were made. To illustrate this the results of one set of readings taken with bright sun in a nearly clear sky are shown in Table IV and a summary of averages is given in Table V. The cardioid type receptor is considered as the reference and the constant C_b for the cosine type is determined for the condition which would make the cosine type give the same indication as the cardioid type receptor. The scale of the exposure meter used was divided into $\frac{1}{3}$ stop intervals and can be read to $\frac{1}{6}$ stop intervals, at the deflections obtained in the test. In consequence the figures shown in cols. 4, 5, 6, 7 of Table IV are expressed in terms of $\frac{1}{6}$ stop intervals, thus 13/1½ represents 13 divisions and 1½ sixths of a stop.

Col. 7 shows the difference (in $\frac{1}{6}$th stops) between and cardioid and cosine receptor readings.

Col. 8 shows the arithmetical value of the stop value given in col. 7. This is worked out on the basis that $\frac{1}{6}$ stop is equivalent to $10^{\frac{1}{20}}$.

Col. 9 is obtained by dividing the constant of the cosine receptor $C_b=51$ by col. 8 and is that constant which (at the angle of γ given) would cause the cosine and cardioid receptors to give the same reading. The cardioid receptor has a constant of $C_a=40$ and in col. 11 is shown the ratio $\frac{C_b}{C_a}$, i.e. $\frac{\text{col. (9)}}{40}$.

Col. 11 shows for an angle $\delta=54°$ the values of the ratio $\frac{C_b}{C_a}$ obtained by interpolation of the figures of Table III.

It will be noticed that the agreement between col. (10) and col. (11) is apparently not close, but if the fact is taken into account that a discrepancy of $\frac{1}{6}$ stop can be considered as reasonable, the agreement is acceptable. Col. (12) shows the ratio col. (10)/col. (11) and these figures may be compared with 1·122, which is equivalent to $\frac{1}{6}$ stop. The conditions of test were not exactly the same as those presumed for the calculation, for there was on all occasions of measurement a small amount of cloud and some haze, and consequently it would be expected (because the actual conditions are more nearly uniform than the assumed conditions) that the measured values would give lower, not higher, readings than the calculated ones. This is not borne out in the test.

Table V. shows a further set of results obtained for tests similar to that already described. For these tests two cosine type receptors were used having constants $C_b=51$ (as above) and $C_b=30$. The readings of the meter and subsequent calculation have been omitted from this tabulation, only the ratio of $\frac{C_b}{C_a}$ is given for the various angles.

The ratio $\frac{C_b}{C_a}$ for angles between 90° and 270° is observed to be nearly constant at the value obtained

Table V

Comparison of cardioid and cosine receptors for conditions of sunrise in a blue sky

1	2	3	4	5	6	7	8	9
Observation	1	2	3	4	5	6	7	8
Cosine receptor constant	51	30	51	30	51	30	51	30
β	49	49	50	50	53	53	42	42
γ	Ratio					$\frac{C_b}{C_a}$ for $C_a = 40$		
0	·827	·917	·980	·915	·960	1·035	·960	·945
30	·855	·917	·980	·945	1·012	1·057	·850	·892
60	·717	·817	·857	·840	·880	·862	·802	·772
90	·552	·545	·637	·630	·637	·670	·637	·595
120	·570	·565	·632	·620	·650	·647	·650	·595
150	·570	·595	·627	·645	·650	·670	·675	·630
180	·582	·630	·620	·670	·677	·647	·760	·647
210	·552	·595	·627	·670	·650	·647	·717	·667
240	·552	·595	·570	·620	·650	·647	·620	·580
270	·535	·560	·580	·620	·600	·630	·650	·647
300	·640	·645	·760	·795	·787	·795	·732	·667
330	·760	·817	·855	·862	·855	·892	·905	·842
360	·827	·892	·880	·945	·932	·945	·957	·892
Average of values for 0–90 and 270–360	·715	·765	·817	·795	·834	·861	·812	·782
Estimated averages	·74	·74	·74	·74	·735	·735	·76	·76

for 90° (or 270°). This is to be expected in view of the fact that the point source is the main contributor to one of the measurements involved, whilst the uniform illumination is the only contributor to the other.

Conditions of uniform illumination in which the meter reading is independent of the angle γ were not often encountered, but a set of readings was obtained in conditions which very nearly resulted in a constant reading for all horizontal directions of the receptor axis. The results of this test are shown in Table VI.

The calculated value of this ratio was 0·66. Here the measured value is less than the calculated value, and because the conditions of test were not as uniform as those presumed in the calculation it would be expected that they would have been greater.

It is evident that from the calculation and measurement that there is no simple single value of the ratio $\frac{C_b}{C_a}$ which will satisfy all photographic conditions. The test results show that the measured values of $\frac{C_b}{C_a}$ are greater than the estimated ones, further it would appear reasonable to bias the choice of this ratio in favour of photographs taken with the lighting most likely to be encountered, that is sunlight for which γ is less than 90°, and to take less account of the requirement of uniform lighting. Thus a higher value of $\frac{C_b}{C_a}$ than that given in Section 5.2 should prove satisfactory and accordingly a ratio $\frac{C_b}{C_a} = 0 \cdot 75$ is chosen.

The actual value for C_a and for C_b is discussed in the next section.

Table VI

Comparison of cardioid and cosine type receptors. Completely overcast sky

1	2	3	4	5	6	7	8	9
Type of receptor	*Constant C*	*Meter Readings*			*Diffce. readings 1/6 stop*	*Factor*	*C′*	$\frac{C'}{40}$
		$a=0$	*all values of γ*	*Average*				
Cardioid	$C_a=40$	—	10/3	—	—	—	—	—
Cosine 1	$C_b=51$	10/1	8/3	9/2	7	2·24	22·7	·568
Cosine 2	$C_b=30$	11/1	9/2	10 1½	1½	1·19	25·2	·63

6. THE VALUES OF THE CONSTANTS C_a AND C_b

It has already been stated that the value of the constant to be used for any one incident light receptor must be determined by photographic tests, that is by finding the conditions under which acceptable pictures are obtained.

A test for this purpose has been carried out and will now be considered. The test was carried out using colour film transparencies.

6.1. Test Conditions

The factors which must be taken into account in this test are:—

(i) the camera relative aperture (A);
(ii) the camera shutter speed (t);
(iii) the film speed (S);
(iv) the exposure meter indication of the illumination E incident upon the subject;
(v) the developing process by which the finished transparency is obtained, and finally
(vi) the acceptability of the final picture, as assessed by a number of observers.

The relative aperture of a camera depends upon its physical dimensions and it can be assumed that the error in it is sufficiently small to be negligible compared with the errors in the other factors listed above. The camera used in this test was an Agfa Super Silette and its shutter speed was measured both before and after the test. During the test all of the exposures made were at a camera shutter speed of 1/60 second, and for this speed setting the measured exposure time was observed to be between 18·5 and 19·5 milliseconds.

The film used was Kodachrome II which has a nominal film speed S=25. The 36-exposure reels were available all having the same batch number, and therefore presumed to be of the same speed.

From the time the reels were received from the supplier they were kept as nearly as possible in the same physical conditions, particularly of temperature. Before processing one reel was exposed on a sensitometer in order to measure the film speed, it being presumed that this was reasonably representative of all the other films of the batch, which were exposed to obtain transparencies.

In order to attain consistency in the development process all used films were kept together (with the unused ones) until the test was completed and arrangements were made for all of them including that for determination of S, to be developed at the same time at the Kodak processing laboratories. The exposure meter used was a special one consisting of a hemispherical incident light receptor, the characteristic of which approached very closely to that of a cardioid, together with a meter. The receptor was attached by a flexible lead to the meter and the combination was calibrated to have a constant C=40. Subsequent measurements on this meter gave values of C of 42·2, 38·3 and 39·2 with average 39·9 showing that the accuracy of determination of the constant may be taken as about ±5 per cent. The calibration marks on the scale were also checked and it was found that no significant error occurred in their relative positions, in other words the uncertainty of measurement by the meter can be taken as not greater than ± 5 per cent, i.e. approximately 1/12 stop.

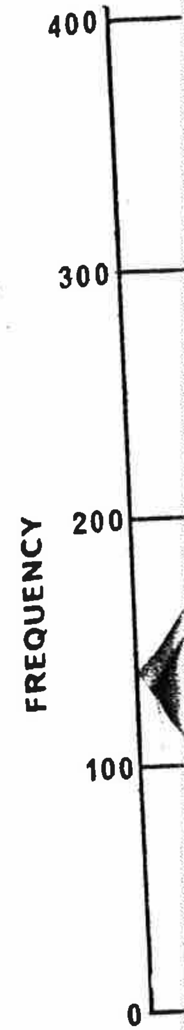

Fig. 3. Freque

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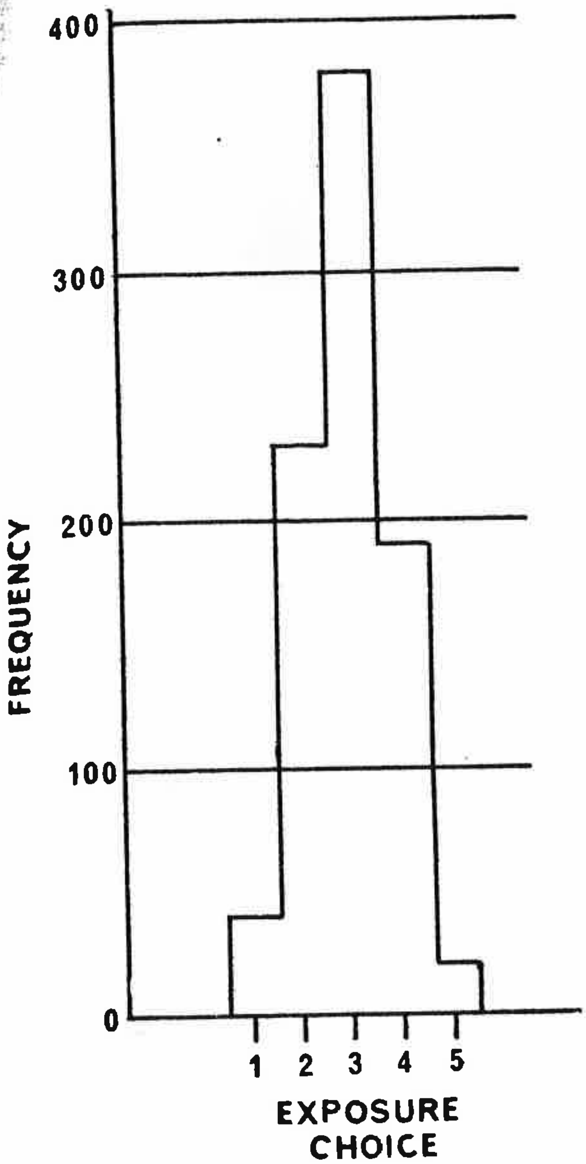

Fig. 3. Frequency distribution of exposure choice for all observations.

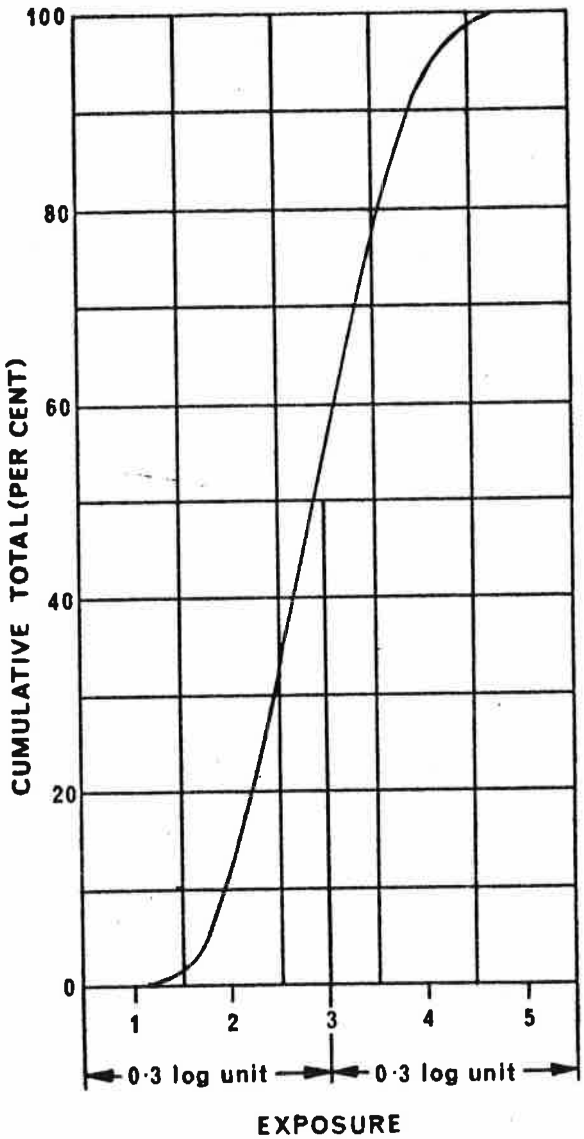

Fig. 4. Ogive derived from Fig. 3.

It is not simple to devise a test which will give an unequivocal answer to the question, "Is the level of exposure used to obtain this picture too much, too little or correct?" This is in effect the question to be answered in order to choose a reasonable value for the constant C, and it was considered that a valid answer can be obtained only by finding which of a number of pictures is the most acceptable when they have been taken at different equally spaced levels of exposure. To do this, five exposures were taken of each scene used and these were made at half stop intervals of exposure from one stop above the exposure indicated by the exposure meter to one stop below. The resulting transparencies were arranged in sequence of either increasing exposure or of decreasing exposure and were shown to a number of observers.

The observers were asked to say which of the pictures shown appeared to them to be most nearly correctly exposed. This they did by recording the number (from 1 to 5) in sequence, which indicated their choice from the five exposures projected. If an observer thought that the ideal would have been between any two of the exposures shown, such for instance as between numbers two and three of the sequence, this was recorded as a choice of 2/3.

For viewing, a beaded screen was used and the observers were located as nearly as possible to a line drawn perpendicularly from the screen centre. Measurements were made, by means of an SEI photometer, of the screen luminance from the positions occupied by the six observers used during each session of the test, when the screen was illuminated by the projector without transparencies. The values of the luminance varied from 20 to 38 ft. lamberts according to the different observer locations, but each observer always occupied the same position relative to the projector throughout the whole test.

The line of sight of the observers was at an angle between 20° and 10° from the perpendicular to the screen.

Photographs were taken "in the field" and resulted in 54 different scenes consisting of five exposures each. For each scene taken the meter reading, the camera setting $f/$ and t, the sun elevation and the sun angle were measured and recorded, and further the time of day was noted. All scenes were within a few miles of each other and exposures were taken as far as possible in bright sunshine. The latitude was approximately 51½°N. Each of 12 observers was shown 48 sets of 5 exposures (48 different scenes) and another 12 observers were shown 24 sets. This resulted in a total of $48 \times 12 + 24 \times 12 = 864$ observations, of which 861 were acceptable and three rejected because of uncertainty of the observer's intention.

6.2. Results of the Photographic Test

6.2.1. *The value of the constants C_a and C_b*

The sequence of numbers (one to five) used to assess each scene was such that number one indicated an exposure one stop more than that given by the exposure meter reading, i.e., over exposure and number five indicated one stop less i.e., under exposure. This condition was maintained throughout the analysis of results although the sequence in which the exposures were projected was approximately fifty per cent from over to under exposure and the other fifty per cent from under to over exposure. This reversal of sequence did not make any significant difference to the choice of the most acceptable picture.

The overall result of all 861 observations is shown in the histogram of Fig. 3, derived from the figures shown in Table VII. In Table VII Column 1 shows the numbers used by the observers to indicate their choice of most acceptable transparency, column 2 shows the frequency with which each possible choice was selected in the classifications actually used by the observers.

It will be seen that the number of observers who used the intermediate classifications 1/2, 2/3, 3/4 or 4/5 is considerably less than the others and accordingly the number of these intermediate choices is shown in column 3 equally divided between those on either side of them, thus 232 is obtained from 172 plus ½ (16+104). By this means the interval in exposure difference is maintained at ½ stop.

If the constant C_a chosen for the meter were correct the histogram of Fig. 3 would be symmetrical about the position 3. This, it is not. It is therefore necessary to find the position about which the histogram is symmetrical. This is most simply done by making a cumulative total of results as shown in column 4 of Table VII and expressing this in terms of percentages (column 5). A curve of these percentages is then drawn (the ogive of Fig. 4) and the position at which this curve crosses the 50 per cent mark corresponds to the exposure interval about which the histogram is symmetrical.

An exposure interval of one stop is equivalent to 0·3 on a $\log_{10}$ scale, and hence the difference in exposure intervals between position 3 (corresponding to $C=40$) and that about which the histogram is symmetrical, can be used to apply a correction to the constant $C=40$.

The above method of analysis has been explained in some detail because it is used henceforth in the further discussion of these results but for reasons which are given later the correction which could be applied from the ogive of Fig. 4 has not been made.

Table VII

Record of all valid observations

1	2	3	4	5
	Frequency		*Cumulative total*	
Exposure choice	*Recorded*	*Adjusted*	*Number*	*Percentage*
1	30	38	38	4·4
1/2	16	—	—	—
2	172	232	270	31·4
2/3	104	—	—	—
3	277	381	651	75·6
3/4	104	—	—	—
4	136	191½	842½	97·9
4/5	7	—	—	—
5	15	18½	861	100

The camera mechanism is of such a nature that its aperture can be set at discrete half-stop intervals whilst the shutter speed can be set at one-stop intervals. The exposure meter on the other hand is a continuously reading device which permits reading of the indication of illumination to within 1/12 stop. The meter reading is thus capable of calling for exposures which cannot be provided by the camera with the result that (independent of the error in shutter speed which is held constant by using the same speed throughout the test), there may be a discrepancy of $\frac{1}{6}$ stop between the camera setting and the required exposure. (An exposure meter reading of say $E_v = 12\frac{1}{6}$ can be reproduced on the camera as $E_v = 12$ and a meter reading $E_v = 12\frac{1}{3}$ can be reproduced as $12\frac{1}{2}$).

To determine the effect of this possible discrepancy an analysis was made in terms of the E_v called for by the meter and that achieved in the camera. The sets of five exposures were grouped in accordance with the difference between the meter and camera E_v values. For 18 sets the exposure meter indication and the camera setting were the same (no error caused by the discrepancy described), for 14 sets the difference was $\frac{1}{6}$ stop tending to produce over exposure and for 6 sets it was $\frac{1}{6}$ stop tending to produce under exposure. The analysis of results for these three groups is shown in Table VIII and as we are interested only in the value of the constant C the ogives are plotted in Fig. 5.

Fig. 4 shows the very interesting result that this manner of test is able to distinguish $\frac{1}{6}$ stop difference in exposure and further that when the exposure meter indication and camera setting are the same, that the chosen value of $C_a = 40$ is correct.

This is not a final conclusion; other errors are to be taken into account. It has been stated that the shutter speed used throughout the test was 1/60 second which implies a nominal exposure time of 16·7 milliseconds. The actual exposure time can

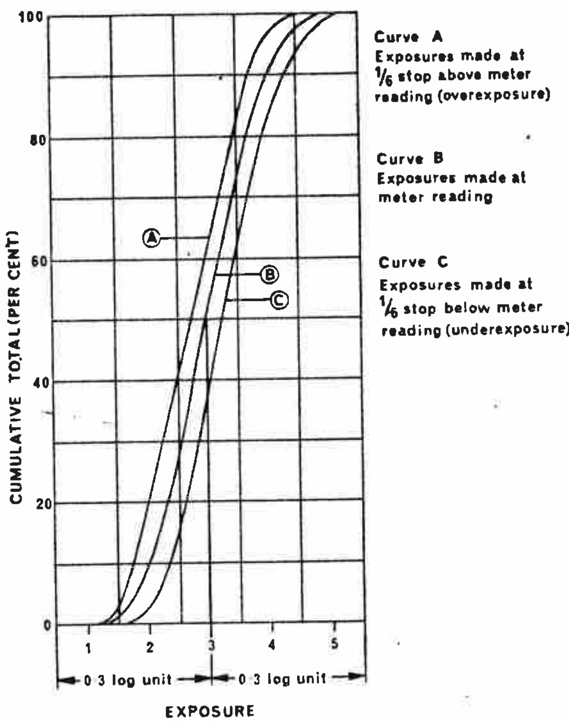

Fig. 5. Analysis of exposure choice in relation to camera setting.

be taken as the mean of the measurement made and was 19 milliseconds. This means that all exposures were increased in the proportion $\frac{19}{16 \cdot 7} = 1 \cdot 14$ and were over exposed by this amount.

The measured film speed was found to be 0·08 log

Table VIII

1	2	3	4	5	6	7	8	9	10
Camera error No. of sets	Zero 18			+1/6 stop 14			−1/6 stop 6		
Exposure choice	*Number*	*Cumulative total*		*Number*	*Cumulative total*		*Number*	*Cumulative total*	
		No.	*Percentage*		*No.*	*Percentage*		*No.*	*Percentage*
1	7	7	2·3	21	21	9·2	2½	2½	2·1
2	85½	92½	29·7	70½	91½	40·1	15½	18	15·4
3	130	222½	71·3	100½	192	84·2	52½	70½	60·2
4	83	305½	97·8	35	227	99·6	37½	108	92·3
5	6½	*312*	100	1	*228*	100	9	*117*	100

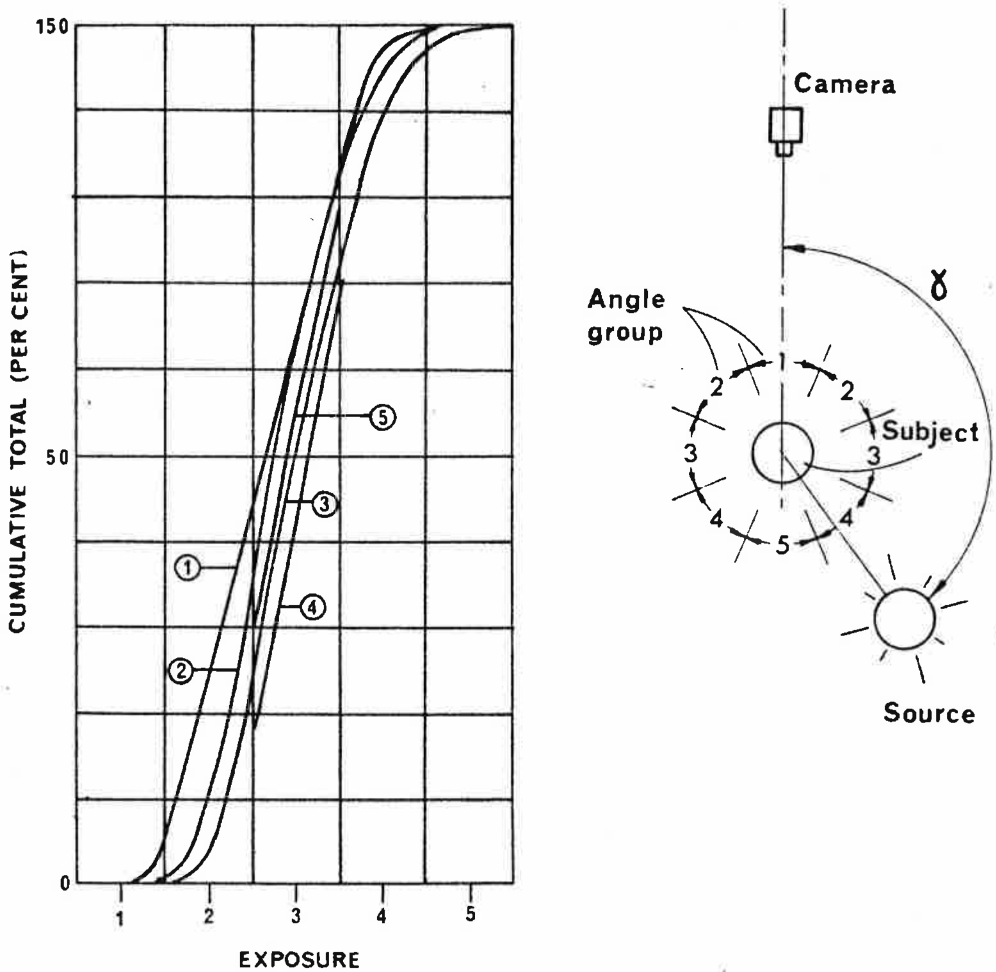

Fig. 6. Analysis of exposure choice in relation to sun angle.

units slow, and this is equivalent to all exposures being under exposed in the proportion antilog ·08=1·20. It is to be noted that this departure from nominal represents only about ¼ stop and is within the manufacturing tolerance for the speed of films.

The curve of Fig. 5 shows that the nominal constant $C_{\varepsilon}=40$, together with the errors mentioned above results in correct exposures and consequently

(i) if the camera shutter speed is corrected the picture would be *less* exposed. This could be corrected by decreasing the meter reading (to give more exposure) which can be obtained by decreasing the optical transmission factor of the incident receptor, which in turn is equivalent to an increase of the constant C_{ε}. In this case C_{ε} would increase to $40-1\cdot14=45\cdot6$.

Table IX

Angle group No. ...	1		2		3		4		5	
Median (degrees) ...	0		45		90		135		180	
Range (degrees) ...	38 to 22		23 to 67 293 to 337		68 to 112 248 to 292		113 to 157 203 to 247		158 to 202	
No. of sets	10		19		9		7		1	
No. of observations	162		305		161		120		24	
Exposure choice	*No.*	*Percent. total*	*No.*	*Percent. total*	*No.*	*Percent. total*	*No.*	*Percent. total*	*No.*	*Percent. total*
1	12½	7·7	20½	6·7	2	1·2	2	1·7	—	
2	59	44·1	92	36·9	35½	23·3	18½	17·1	7½	31·2
3	59½	80·9	137½	82·0	78½	72·0	64½	70·8	11½	79·2
4	31	100	53	99·4	41½	97·8	33½	98·8	5	100
5	—		2	100	3½	100	1½	100		
Total	162									

(ii) if the film speed is corrected the picture would be *more* exposed. This could be corrected by increasing the meter reading (to give less exposure) which can be obtained by increasing the optical transmission, equivalent to a decrease in constant C_a. Combining the two effects, the decrease is to $\frac{45 \cdot 6}{1 \cdot 20} = 38$.

This figure is however subject to further qualification.

There is, in all the measurements involved in determination of this figure, some uncertainty, and we cannot be sure that the corrections applied are at the mean point within the range of uncertainty. It has already been pointed out that the uncertainty in calibration of the meter is of the order of ± 5 per cent. The accuracy of the shutter speed measurement from the figures quoted is not better than ± 3 per cent and the film speed probably cannot be estimated to better than 0·02 log units, equivalent approximately to $\pm$ 5 per cent. To allow for these conditions it is evident that more information is required than is at present available and a greater tolerance in C is desirable than need be applied to the constant K for the reflected light method.

It is therefore suggested that the range for the constant C_a should be 30 to 50; it will be noted that this represents a greater tolerance for the high values of C_a ($50 = 38 \times 1 \cdot 32$ whereas $30 = \frac{38}{1 \cdot 26}$ but experience shows that where the pick-up characteristic of the receptor departs from the cardioid a higher than nominal value of C_a is more likely to be satisfactory than a low one.

For the cosine type receptor the range of the constant C_b will then be ·75 times that of the constant C_a, i.e. $C_b = 22 \cdot 5$ to $37 \cdot 5$ with a nominal value of 28·5.

6.2.2. *The validity of the cardioid type receptor*

It has been stated that the cardioid type receptor can provide satisfactory exposure determination by a single meter reading and this can only be true if the exposures obtained by accepting the readings of a cardioid type meter are satisfactory independently of the relative position of subject and sun. In the test carried out the sun angle was measured for each scene photographed clockwise from the line between subject and camera, thus the angles 45° and 315° both represent a 45° sun angle, and similarly 90 and 270 represent 90° and 135 and 225 represent a 135 sun angle. The sets of exposures were selected in accordance with this grouping of angles using every 45° interval as the median of the group, and an analysis similar to that already described was carried out. The results are shown in Table IX in which for each angle range used the progressive total of observations is given, and its percentage value.

The ogives of Fig. 6 show these results. The intercept of the curves with the 50 per cent line occurs at points nowhere in excess of ·045 log units equivalent to about ⅙ stop away from the central log value which represents correct exposure. In other words the cardioid type receptor can be expected to give correct exposure indication independently of the relative direction of the light on the subject.

Because the relationship between the cosine and the cardioid type receptors, as shown in Table III varies by nearly one stop when the angle γ varies from 0 to 90° the cosine type receptor will give performance varying by a similar amount relative to that of the cardioid type receptor. As the cardioid type receptor gives correct exposure indication within about $\frac{1}{6}$ stop for all values of γ, as shown by Fig. 6, the cosine type receptor will be in error by the amount of its variation relative to the cardioid type. At best this will be of the order of $\pm\frac{1}{2}$ stop.

7. CONCLUSION

It is appreciated that the amount of evidence which it has been possible to obtain and analyse in the preparation of this report is not sufficient to establish the values of the constant C for cosine and cardioid type receptors without any further question, but it can be accepted that the cosine type receptor cannot provide the same performance as the cardioid receptor under all conditions of its use; that is, by taking readings with the receptor directed from the subject to the source of illumination or directed from the subject to the camera, or by taking the logarithmic average of the readings obtained by the above two methods. This is proved both by the preliminary investigation into the problem and by the experimental evidence, which whilst not in exact agreement with the former, nonetheless bears out its findings by showing the same type of relative relationship between the cosine and cardioid receptors.

The cardioid receptor is shown to be capable of giving correct exposure information within about $\frac{1}{6}$ stop for any sun angle. This implies that the use of a single reading obtained with this type of receptor, is valid. A satisfactory value of the constant C_a for the cardioid receptor would appear to be 38 when illumination is measured in lumens/sq. ft. (equivalent to 410 when illumination is measured in lux), but a wide limit has been suggested for this constant because firstly, of the possibility that further tests may reveal the necessity to make some small adjustment to it, and secondly of the fact that when the cardioid characteristic is not perfectly achieved it is still possible to obtain a receptor which will give satisfactory photographic results but this demands a different (and usually higher) value of the constant.

Because of the limitations of the performance of the cosine type receptor any one value of its constant can be shown to be invalid under certain conditions of use. If it be assumed that the cosine receptor will be used mostly in conditions of sunlight with the sun angle not exceeding about 60° which will be a frequent condition of use, a constant of C_b of 80 to 85 per cent of that of C_a is admissible (see Table V) i.e. C_b= 30·5 to 32·5, but a figure of 75 per cent should provide a wider field of application. This latter figure gives C_b=28·5 equivalent to 307 when the unit of illumination is the lux. There is little indication as to which of these figures is to be preferred, and in consequence a wide range of permissible values has also been suggested for C_b.

The problem here dealt with has been investigated in the detail shown in this report in order to attempt to establish the bases of incident light exposure determination and prove the merit of the cardioid receptor relative to that of the cosine receptor. The practical evidence is necessarily limited by the location of the observer and justifies the theoretical considerations which enable the problem to be seen in a wider context than the evidence permits. There remains only the necessity for independent practical verification of the conclusions reached, by further photographic testing.

ACKNOWLEDGEMENT

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APPENDIX A
THE COSINE TYPE RECEPTOR

1. Definition

The cosine type receptor can be represented by the equation

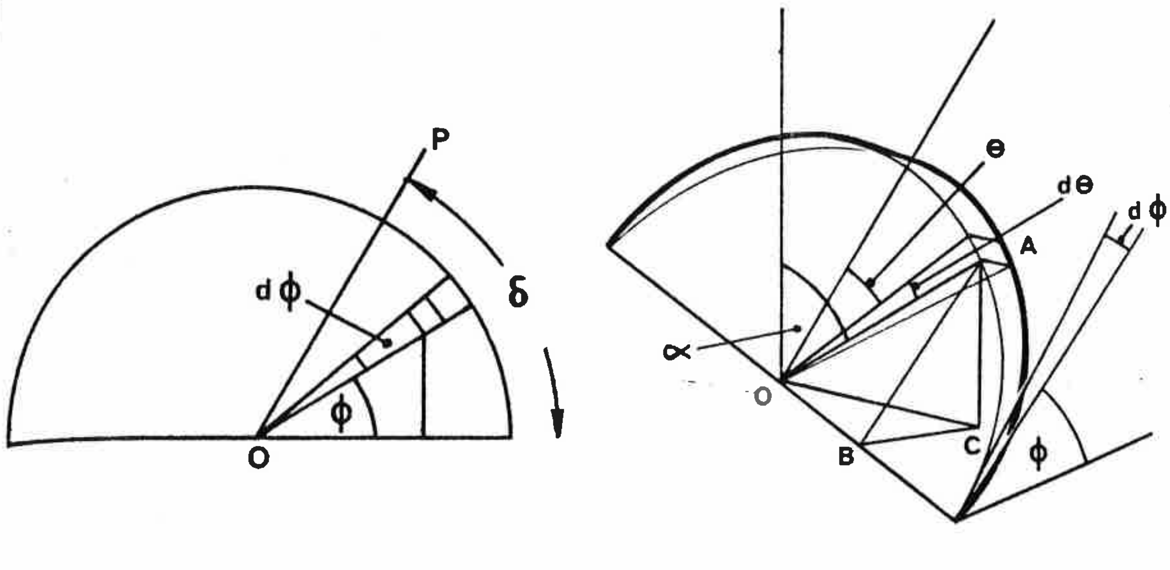
$$R_b = k_b\, E \cos \alpha \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad A(1)$$

where R_b is the response of the meter to the illumination on the receptor and is the exposure meter indication which is transferred to the calculator mechanism to obtain the exposure required.

k_b is a constant of proportionality which takes into account the size and optical transmission characteristics of the receptor.

E is the illumination on a plane perpendicular to the source axis provided by the source of light at the position occupied by the receptor.

α is the angle between the receptor axis (the normal to the receptor) and the line joining the centre of the receptor to the centre of the source of light.



A.1.a. **A.1.b.**

Fig. A.1. Analysis of receptor response with source of uniform luminance.

2. Cosine Receptor Subjected to Illumination from a Point Source

If the source has luminance L_1 and surface area S_1 its candle power is L_1S_1. If it is small enough to be regarded as a point source, the illumination produced by it at distance r is $E_1 = \frac{L_1S_1}{r^2}$.

The response of the meter is

$$R_{b1} = k_b \frac{L_1S_1}{r^2} \cos \alpha \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad A(2)$$

3. Cosine Receptor in a Sphere with Internal Surface of Uniform Luminance

Assume that the light receptor is placed at the centre of a sphere of uniform internal luminance L_2. The sphere is assumed to have a large radius compared with the dimensions of the light receptor so that all parts of the receptor receive the same illumination.

Consider an elemental surface δS of the sphere, which has radius r contained between four planes, two of which intersect along a diameter of the sphere and are inclined to each other at angle $d\phi$, and the other two perpendicular to the first two and containing the angle $d\theta$. Refer to Fig. A.1 a and b.

The area of the element is $\delta S = r d\theta \times r \cos\theta \, d\phi$ and its apparent candle power is $\delta I = L_2 r^2 \cos\theta \, d\theta \, d\phi$. This produces an illumination dE at 0, the centre of the sphere where

$$dE = \frac{\delta I}{r^2} = L_2 \cos\theta \, d\theta \, d\phi$$

The receptor is assumed to be placed in the reference (horizontal) plane with its axis vertically upwards and

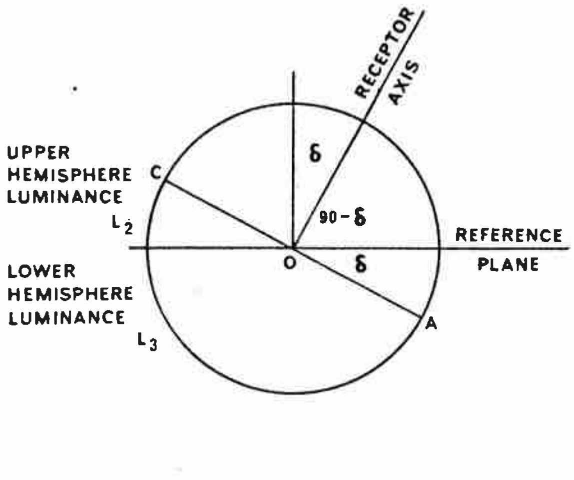


Fig. A.2. Cosine receptor in hemispheres of differing uniform luminances.

it is, therefore, inclined at an angle a to the direction of the light from the element δS, where

$$\cos a = \frac{AC}{OA} = \frac{AB \sin \phi}{r} = \cos \theta \sin \phi.$$

The response of the receptor to light from the element is

$$dR_b = k_b L_2 \cos \theta \, d\theta \, d\phi \cos a$$
$$= k_b L_2 \sin \phi \, d\phi \cos^2 \theta \, d\theta.$$

The total effect of the elemental strip of the sphere surface between the two planes

$$\text{is } 2 \int_{\theta=0}^{\theta=\frac{\pi}{2}} dR_b = 2k_b L_2 \sin \phi \, d\phi \int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$$

$$= 2k_b L_2 \sin \phi \, d\phi \left[\tfrac{1}{2} \sin \theta \cos \theta + \tfrac{1}{2} \theta \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} k_b L_2 \sin \phi \, d\phi$$

The effect of that part of the sphere enclosed between the reference plane and the plane OP at angle δ to the reference plane is then

$$\frac{\pi}{2} k_b L_2 \int_0^{\delta} \sin \phi \, d\phi = \frac{\pi}{2} k_b L_2 (1 - \cos\delta) \quad \ldots \quad A(3)$$

The total effect R_{b2} of the sphere is in this case the same as that of the upper hemisphere in the diagram Fig. A1.a because the response of the receptor is zero for light received from the lower hemisphere. It is found by putting $\delta = \pi$ whence

$$R_{b2} = \pi k_b L_2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad A(4)$$

4. Cosine Receptor in a Sphere with Hemispheres of Differing Luminances

The upper hemisphere is presumed to have luminance L_2 and the lower hemisphere luminance L_3.

The axis of the cosine receptor is inclined at an angle $(90 - \delta)$ to the horizontal or reference plane. Refer to Fig. A.2.

The response of the meter to the luminance L_3 of the lower hemisphere (OAB) is given from equation A_3.

$$R_l = \frac{\pi}{2} k_b L_3 (1 - \cos \delta)$$

For the upper hemisphere (OBC) the response is found by substituting $(\pi - \delta)$ for δ in the above equation, and is

$$R_u = \frac{\pi}{2} k_b L_2 (1 + \cos \delta)$$

The total response

$$R_{b3} = R_l + R_u = \frac{\pi}{2} k_b \left\{ L_2 (1 + \cos \delta) + L_3 (1 - \cos \delta) \right\}$$

$$= \frac{\pi}{2} k_b (L_2 + L_3) + \frac{\pi}{2} k_b (L_2 - L_3) \cos \delta$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (A5)$$

In the special condition when the receptor axis is horizontal, the response is obtained from half the light of the upper hemisphere plus half that from the lower.

In this case $\delta = 90°$ and

$$R_{b3} = \frac{\pi}{2} k_b (L_2 + L_3) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad A(6)$$

APPENDIX B
THE CARDIOID TYPE RECEPTOR

1. Definition

The cardioid type receptor can be represented by the equation

$$R_a = \tfrac{1}{2} k_a E (1 + \cos a)$$

in which the symbols have similar significance to those used in Appendix A. The constant of proportionality k_a is not necessarily of the same value as k_b.

2. Cardioid Receptor Subjected to Point Source Illumination

If the same assumptions are made. as were used in Appendix A. the response is given by

$$R_{a1} = \tfrac{1}{2} k_a \frac{L_1 S_1}{r^2} (1 + \cos a) \quad \ldots \quad \ldots \quad \ldots \quad B(1)$$

3. Cardioid Receptor in a Sphere of Uniform Luminance

If the conditions assumed for the cardioid type receptor are the same as those for the cosine type given in Appendix A, the illumination produced by the element of surface at the centre of the sphere has been shown to be

$$dE = L_2 \cos\theta \, d\theta \, d\phi$$

The receptor is assumed to be placed as before with its axis vertically upwards and its inclination to the light from the element δS is again α, where $\cos\alpha = \cos\theta \sin\phi$. The response of the receptor to light from δS is then

$$dR_a = k_a L_2 \cos\theta \, d\theta \, d\phi \, \tfrac{1}{2}(1+\cos\alpha)$$

$$= \tfrac{1}{2} k_a L_2 (\cos\theta + \cos^2\theta \sin\phi)\, d\theta \, d\phi$$

The response to light received from the elemental strip between the two planes when the strip lies above the reference plane (ϕ is positive) is:—

$$2\int_{\theta=0}^{\theta=\pi_2} \tfrac{1}{2} k_a L_2 (\cos\theta + \cos^2\theta \sin\phi)\, d\theta \, d\phi$$

$$= k_a L_2 \left(1+\frac{\pi}{4}\sin\phi\right) d\phi$$

and from the whole of the upper hemisphere is:—

$$k_a L_2 \int_{\phi=0}^{\phi=\pi} \left(1+\frac{\pi}{4}\sin\phi\right) d\phi = \frac{3}{2}\pi k_a L_2$$

The response to light received from an elemental strip when this lies below the reference plane (ϕ is negative) is:—

$$2\int_{\frac{\pi}{2}}^{\pi} \tfrac{1}{2} k_a L_2 \left\{\cos\theta + \cos^2\theta \sin(-\phi)\right\} d\theta \, d\phi$$

$$= -k_a L_2 \left(1+\frac{\pi}{4}\sin\phi\right) d\phi$$

and from the lower hemisphere is:—

$$= k_a L_2 \int_{0}^{-\pi} \left(1+\frac{\pi}{4}\sin\phi\right) d\phi = \frac{\pi}{2} k_a L_2$$

The total response of the cardioid receptor to light from the whole of the sphere is

$$R_{a2} = \left(\frac{3}{2}+\tfrac{1}{2}\right)\pi k_a L_2 = 2\pi k_a L_2 \quad \ldots \quad \ldots \quad B(2)$$

4. Cardioid Receptor in a Sphere with Hemispheres of Differing Luminances

The upper hemisphere is of luminance L_2 and the lower of luminance L_3. The axis of the receptor is assumed to be along a horizontal line so that one half of the receptor may be considered to be influenced by the upper hemisphere and the other half by the lower. In this case the total effect on the receptor is

$$\tfrac{1}{2}\cdot 2\pi k_a L_2 + \tfrac{1}{2}\, 2\pi k_a L_3 = \pi k_a (L_2 + L_3) \quad \ldots \quad B(3)$$

The condition investigated in Appendix A in which the receptor axis is inclined at an angle $(90-\delta)$ to the reference plane is not of interest for the cardioid receptor, for in its use it is directed from the subject towards the camera which in the simplification made here is along a horizontal line.

APPENDIX C

DAYLIGHT CONDITIONS

In order to determine reasonable values to be assigned to the quantities given in the various equations used in the text, reference is made to the Smithsonian Physical Tables from which the following information is obtained:—

p.339.	Brightness of sun as observed at earth's surface ...	165000 cd/cm²
p.339.	—clear sky (average) ...	0·4 cd/cm²
p.623.	Diameter of sun	$1{\cdot}391\times10^{6}$ km
p.601.	Mean distance earth to sun	$1{\cdot}495\times18^{8}$ km

If the point source referred to in Appendices A and B is the sun,

$$L_1 = 165000 \text{ cd/cm}^2$$

$$S_1 = \frac{\pi}{4}(1{\cdot}391)^2 \times 10^{12} \text{ km}^2$$

$$r = 1{\cdot}495\times10^{8} \text{ km}$$

so that $\dfrac{L_1 S_1}{r^2} = 11{\cdot}219$ lumens/cm².

If the luminance of the upper hemisphere referred to in Appendices A and B is obtained from the clear sky

$$L_2 = 0{\cdot}4 \text{ cd/cm}^2.$$

and it remains to find a reasonable value for the luminance of the lower hemisphere.

It is known that the effect on a light receptor subjected to the light from an infinite plane of uniform luminance is the same as that of a hemisphere of the same luminance. It is then permissible to assume that the lower hemisphere of Appendices A and B can be used as a reasonable representation of the flat earth.

In the simplification here devised, the earth is presumed to be represented by an infinite matt surface plane of average reflectance p which has a uniform luminance L_3.

Since the clear sky, or upper hemisphere, has a luminance L_2, the lower hemisphere will have a luminance pL_2 produced by the upper hemisphere.

The illumination produced by the sun on the horizontal plane is $\dfrac{L_1 S_1}{r^2}\cos\delta$ where $\beta=(90-\delta)$ is the sun elevation, and hence the reflected illumination will be $p\frac{L_1S_1}{r^2}\cos\delta$ which could be presumed to be provided from a sphere having luminance

$$p\frac{L_1S_1}{\pi r^2}\cos\delta$$

Thus the total luminance of the lower hemisphere is:—

$$L_3=p(L_2+\frac{L_1S_1}{\pi r^2}\cos\delta) \quad \ldots \quad \ldots \quad \ldots \quad C(1)$$

which for the condition of sun in a clear sky and an allowance of 16 per cent for the average reflectance gives—

$$L_3=0\cdot16\,(0\cdot4+\frac{11\cdot219}{\pi}\cos\delta)$$

$$=(0\cdot064+0\cdot571\cos\delta)$$

If the sky is completely overcast the sun being no longer visible, the upper hemisphere may be presumed to have a luminance qL_2 where q is a numeric which takes into account the possibility that the sky may be covered by brilliant white clouds or the sun may have set and the resultant luminance be very low. It is perhaps fortunate that the simplifications here assumed make it unnecessary to allocate a value to the numeric q; alternatively to speculate that the difficulty of allocating this value may be presumed to be the deciding factor in limiting the extent to which the simplifications made may approach the actual conditions of photography. In this case the lower hemisphere will have luminance $L_3=pq\,L_2$.

APPENDIX D
INCIDENT LIGHT MEASUREMENTS

1. Meter Readings when Using a Cosine Type Receptor

The method of use of the cosine type receptor is to take the geometric mean of two readings. The first reading is taken with the receptor axis directed to the source of maximum illumination, the second with it directed from the subject towards the camera.

1.1 *Assume the sun in a clear sky*

The position of the sun relative to the subject is defined by using the line between subject and camera as the reference line. Refer to Fig. 1. The sun elevation is $\beta=(90-\delta)$, and the horizontal angle is γ. The angle γ is the angle in the horizontal or reference plane between the reference line OA and the projection CO on the horizontal, of the line joining subject O and source B. The angle α, of incidence of the light from O to the receptor when the axis of the latter is along OA is given by $\cos\alpha=\frac{OA}{OB}=\frac{OC\cos\gamma}{OB}$ $=\cos(90-\delta)\cos\gamma=\sin\delta\cos\gamma$.

Two readings are taken

(i) With the receptor pointed to the maximum source.

In this case the angle of incidence α is zero and the response of the meter to light from the point source is

$$R_{b1}=k_b\frac{L_1S_1}{r^2}$$

With the receptor axis directed to the source the receptor is illuminated partly from the upper and partly from the lower hemisphere as shown by equation A(5).

$$R_{b3}=\frac{\pi}{2}k_b\,(L_2+L_3)+\frac{\pi}{2}k_b\,(L_2-L_3)\cos\delta$$

so that the total response of the meter is

$$R_{bs}=k_b\left\{\frac{L_1S_1}{r^2}+\frac{\pi}{2}(L_2+L_3)+\frac{\pi}{2}(L_2-L_3)\cos\delta\right\}$$

in which L_3 is as given by equation C(1)

$$L_3=p(L_2+\frac{L_1S_1}{\pi r^2}\cos\delta)$$

When the values given in Appendix C are substituted in this equation:—

$$R_{b1}=11\cdot219\,k_b \qquad L_3=(0\cdot064+0\cdot571\cos\delta)$$

from which

$$R_{b3}=\frac{\pi}{2}k_b\,(0\cdot464+0\cdot571\cos\delta)$$

$$+\frac{\pi}{2}k_b\,(0\cdot336\cos\delta-0\cdot571\cos^2\delta)$$

$$R_{bs}=R_{b1}+R_{b3}=(11\cdot948+1\cdot425\cos\delta$$
$$-0\cdot897\cos^2\delta)\,k_b \quad \ldots \quad \ldots \quad \ldots \quad D(1)$$

(ii) With the receptor pointed towards the camera. To light from the point source the response is

$$R_{b1}=k_b\frac{L_1S_1}{r^2}\cos\alpha=k_b\frac{L_1S_1}{r^2}\sin\delta\cos\gamma$$

The response to the upper and low hemispheres given by equation A(6) is

$$R_{b3}=\frac{\pi}{2}k_b\,(L_2+L_3)$$

which gives a total response

$$R_{bc}=R_{b1}+R_{b3}=k_b\frac{L_1S_1}{r^2}\sin\delta\cos\gamma$$

$$+\frac{\pi}{2}k_b\,(L_2+L_3)=(11\cdot219\sin\delta\cos\gamma$$

$$+\cdot729+\cdot897\cos\delta)\,k_b \quad \ldots \quad \ldots \quad \ldots \quad D(2)$$

The value to be used in determination of exposure is the geometric mean

$$R_b=\sqrt{R_{bs}\times R_{bc}}$$

It is to be noted that in equations D(1) and D(2) when the angle γ is greater than 90° the terms involving $\cos\gamma$ must be taken as zero (they do not become negative), for the receptor does not pick up light from angles greater than when $\gamma=90°$.